Electrical Power and Energy Systems 42 (2012) 299–305

Contents lists available at SciVerse ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Coordination of directional overcurrent relay using evolutionary algorithm and linear programming

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ARTICLE INFO

Article history:
Received 28 November 2006
Accepted 11 March 2012
Available online 16 May 2012

Keywords:
Directional overcurrent relays
Evolutionary algorithm
Linear programming
Optimization techniques
Power system protection
Protection coordination

ABSTRACT

In this article, we present a new method to coordinate the directional overcurrent relay (DOCR) installed in a meshed electricity network. Using evolutionary algorithms and linear programming we solve the problem that allows the calculation of the adjustment intensity (relay setting current, J) and the time multiplier factor, K, such that, in the light of any triphasic or biphasic failure that may occur in the network, the relays may act in the least time possible and in a coordinated manner. We are considering the problem without taking into account the intensity variations that occur when a switch is opened. It may happen that the problem at hand does not have a solution, in that case we determine the constraints that should be removed in order to achieve at least a partial coordination of the relays.

1. Introduction

Directional overcurrent relays (DOCR) are often used as primary protection in distribution networks (normally radial) and as secondary protection in Transmission networks (normally meshed).

The determination of the DOCR adjustment parameters, in such a way that the primary relay and backup relay coordinate correctly, is relatively easy when it is a radial network. On the other hand, when the network is made up of several meshes, the determination of said parameters is a more complicated task.

The methods employed up till now can be classified into the following blocks:

- ☐ Topological analysis
- ☐ Linear programming
- ☐ Non-linear programming
- ☐ Genetic algorithms

The first ones are heuristic methods that determine the relays that open the highest number of loops. The K parameter of all the DOCR of the network is calculated from those relays, in a sequential and recurrent manner. J is considered constant in those methods [1–5].

Methods based on linear programming consider J as constant and suggest a simplified [6] and [7] linear model. Some authors suggest the use of Gauss–Seidel for calculating said K.

Methods based on [12] and [13] non-linear programming determine both K and J of each DOCR of the network. And the problem set out in [6] and [7] is solved using non-linear programming techniques.

The non-linear problem set out in [6] and [7] is solved with genetic algorithms through a genetic algorithm in which both J and K are codified [8–10].

Ref. [11] is an analysis of the different methodologies employed to solve the problem of the coordination of the DOCR of the system, up to the date of publication of the paper.

In this paper, Js and Ks of the DOCR of the system are determined through evolutionary algorithms, in which Js become part of the codification of individuals and Ks are obtained through linear programming as an optimal solution for that J group, based on the optimization criterion. An improvement of this method, as against that suggested by other authors, is that should the problem of optimization not have an overall solution, a systematic method of eliminating restrictions is proposed. The elimination of restrictions means that a linear stretch, protected by the relays implicated in the restriction that has been eliminated, would not have backup protection, even though it would have primary protection. Therefore, the solution to the problem must be found eliminating the least possible number of restrictions.

The advantage that this method has over the one proposed in [10] is that, for each combination of Js obtained with the evolutionary algorithm, the combination of optimum K is obtained by

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http://dx.doi.org/10.1016/j.ijepes.2012.03.036

applying linear programming. Thus the search parameters are reduced by half.

2. Statement of the problem

2.1. Relay operating-time (t)

The operation time of a DOCR with inverse characteristic is given by [14]:

$$t = \frac{a \cdot K}{\left(\frac{I}{J}\right)^{b} - 1} \tag{1}$$

where t is the operation time, when I is constant; a, b is the are relay constants; K is the time multiplier setting; I is the relay current; J is the relay setting current.

The a and b constants depend on the type of characteristics selected: Standard Inverse (SI), Very Inverse (VI) or Extremely Inverse (EI).

2.2. Coordination of a pair of relays

Fig. 1 shows a stretch of network in default.

The group of (i,j) relay pairs are designated by R where i is j's relay backup when an f_j fault occurs in the line protected by j.

In order for the j and i relays to coordinate when an f_j fault occurs, the operation time of the i relay, $t_{ij1}^{f_j}$, must have a time-delay, $r_{(i,j)}^{f_j}$, on the operation time of the j relay, $t_{jj1}^{f_j}$. Mathematically speaking:

$$t_{ij1}^{f_j} - t_{jj1}^{f_j} \geqslant r'_{(i,j)} \quad \forall (i,j) \in R, \quad \forall f_j \tag{2}$$

where f_j is the fault in the line protected by the main relay, j; $t_{jj1}^{f_j}$ is the operation time of j relay when an f_j fault occurs in the line protected by the j relay while the switch for another main relay of said line, k, is closed (1); $t_{ij1}^{f_j}$ is the operation time of i relay when an f_j fault occurs in the line protected by the j relay while the switch for another main relay of said line, k, is closed (1); $r'_{(i,j)}$ is the time-delay between the i and the j relays.

2.3. Posing of the Problem

When there is a fault in one of the lines of the network, it is convenient that: (a) the operation time of the relays be as little as possible, (b) the main relays be the first to operate and (c) only the main relays operate. Therefore, for the f_j fault depicted in Fig. 1, the j and k relays must be the first and only ones to operate, and their operation time, t_j and t_k, must be as little as possible. The problem would be expressed as such:

$$\psi = \sum_{j=1}^{n} \sum_{f_j=1}^{n_{ccj}} t_{jj1}^{f_j}$$
$$\text{Sujeto} \quad a:$$
$$t_{ij1}^{f_j} - t_{jj1}^{f_j} \geqslant r'_{(i,j)} \quad \forall (i,j) \in R, \quad \forall f_j \tag{3}$$

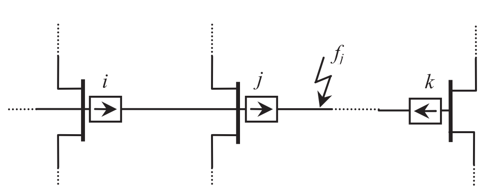

Fig. 1. Pair of directional relays. For the f_j fault the j relay is the main relay and the i is its backup.

where n is the number of relays in the network and n_{ccj} is the number of faults in the line protected by main relay j.

The operation time of each DOCR is determined with the understanding that the I intensity is constant, from the moment the fault occurs until it is isolated.

The adjustment limits of J, J_{min} and J_{max}, depend on the nominal intensity of the line that contains the relay, the adjustment range of said parameter in the relay, the winding ratio of the intensity of the transformer that interconnects the relay with the line and the percentage of the adjacent line that we wish to cover with the relay when it operates as backup. The J limits are those outlined in (4).

$$J_{min} > I_n$$

$$J_{max} = \begin{cases} J_{min} & si \quad \exists \quad I_{cc}^{f} < J_{min} \\ min\{I_{cc}^{f}\} \quad \forall f \in F & si \quad I_{cc}^{f} > J_{min} \end{cases} \tag{4}$$

where I_{cc}^{f} is the intensity that runs through the backup relay when an f fault occurs.

Substituting (1) in (3) and adding the restrictions of K, [K_{min}, K_{max}], and of J, [J_{min},J_{max}] margins, we have:

$$\psi = \sum_{j=1}^{n} \sum_{f_j=1}^{n_{ccj}} \frac{a \cdot K_j}{\left(\frac{I_{jj1}^{f_j}}{J_j}\right)^{b} - 1}$$

$$\text{Sujeto} \quad a:$$

$$\begin{cases} \frac{a \cdot K_i}{\left(\frac{I_{ij1}^{f_j}}{J_i}\right)^{b} - 1} - \frac{a \cdot K_j}{\left(\frac{I_{jj1}^{f_j}}{J_j}\right)^{b} - 1} \geqslant r'_{(i,j)} \quad \forall (i,j) \in R, \quad \forall f_j \\ K_{imin} \leqslant K_i \leqslant K_{imax} \quad \forall i \\ K_{jmin} \leqslant K_j \leqslant K_{jmax} \quad \forall j \\ J_{imin} \leqslant J_i \leqslant J_{imax} \quad \forall i \\ J_{jmin} \leqslant J_j \leqslant J_{jmax} \quad \forall i \end{cases} \tag{5}$$

where $I_{jj1}^{f_j}$ is the intensity that runs through the j relay when an f_j fault occurs in the line protected by the j relay while the switch for the other main relay of said line, k, is closed (1) and $I_{ij1}^{f_j}$ is the intensity that runs through the i relay when an f_j fault occurs in the line protected by the j relay while the switch for the other main relay of said line, k, is closed (1).

When the intensity that runs through the i relay at the time of the f_j fault $I_{ij1}^{f_j}$ is less than or equal to the lower limit of the adjustment intensity, J_i, said relay does not operate (infinite operation time). In such cases, the first restriction of (5) is not included for said faults.

Since the objective function, ψ, and the first restriction are non-linear, the problem is difficult to solve. Its settlement may be tackled with non-linear programming techniques, but their application would be limited by the dimension of the network. To work out any network, it is proposed that an evolutionary algorithm be used to determine the adjustment intensities, J, and to calculate Ks as part of the cost function through linear programming.

The genetic algorithm employed to work out the problem in (5) is based on an evolution strategy $(\lambda + \mu)$-EE, in which only the adjustment intensities, J, of each network relay are codified.

The evolutionary algorithm undergoes an evolution in each repetition, from an initial randomly selected J population, and obtaining, through the use of crossing, mutation and selection operators, better individuals than those of the previous repetition.

Below is a description of elements and operators of the proposed algorithm.

2.3.1. Startup

The process begins by randomly generating a population of μ individuals. Each one of those individuals is codified through a chromosome of actual numbers. For its part, each chromosome is composed of one sequence of pairs or genes, every one of which is formed by the J of each relay and by the standard deviation associated, σ, which will be used in the mutation operator, see Fig. 2. The position occupied by the J in the chromosomes of each relay is fixed.

2.3.2. Crossing

It recombines the genes of two individuals in order to generate two new ones. The cross operator used is bi-sexual, see Fig. 3, which uses two individual *parents* to produce two new individual *children*, both of which bear the characteristics of both parents: Each child inherits genes of J and σ of one of the parents.

2.3.3. Mutation

This operator is applied to each gene of the chromosome, and its aim is to guarantee the diversity of individuals to prevent falling in local minimums.

The operator used is of the adapting type. To calculate the new values of J in the t repetition, $J_i^{(t}$, the standard deviation $\sigma_i^{(t}$ associated with it must first be calculated following the formula below:

$$\sigma_k^{(t} = \sigma_k^{(t-1} e^{(z_o^{(t} + z_i^{(t})} \tag{6}$$

where $z_0^{(t}$ is a random value of normal distribution with zero average and τ_0 standard deviation and $Z_i^{(t}$ is a random value of normal distribution with zero average and τ_i standard deviation.

$$z_0^{(t} \sim N(0, \tau_0^2) \quad z_i^{(t} \sim N(0, \tau_i^2) \tag{7}$$

In these expressions the values τ_0 and τ_i are constants, $z_0^{(t}$ is calculated in each repetition for each one of the individuals, and $z_i^{(t}$ is calculated for each gene of the chromosome.

With the value of $\sigma_i^{(t}$ obtained, $\cup_i^{(t}$ is calculated using the expression below.

$$\upsilon_i^{(t} \sim N(0, (\sigma_i^{(t})^2) \tag{8}$$

The value obtained is added to the value of the adjustment intensity of the previous repetition, $J_i^{(t-1}$.

$$J_i^{(t} = J_i^{(t-1} + \upsilon_i^{(t} \tag{9}$$

Subsequently, it is confirmed that the values of the adjustment intensity $J_i^{(t}$ are within the permissible limits, seen in (4). If not, they are given the corresponding value of the limit (if $J_i^{(t} > J_{imax}$ then $J_i^{(t} = J_{imax}$, if $J_i^{(t} < J_{imin}$ then $J_i^{(t} = J_{imin}$).

J_1^k	J_2^k	J_3^k	…	…	…	J_r^k
σ_1^k	σ_2^k	σ_3^k	…	…	…	σ_r^k

Fig. 2. Chromosome.

Parent 1

J_{11}^k	J_{21}^k	J_{31}^k	…	…	…	J_{r1}^k
σ_{11}^k	σ_{21}^k	σ_{31}^k	…	…	…	σ_{r1}^k

Child 1

J_{11}^k	J_{22}^k	J_{31}^k	…	…	…	J_{r2}^k
σ_{11}^k	σ_{22}^k	σ_{31}^k	…	…	…	σ_{r2}^k

Parent 2

J_{12}^k	J_{22}^k	J_{32}^k	…	…	…	J_{r2}^k
σ_{12}^k	σ_{22}^k	σ_{32}^k	…	…	…	σ_{r2}^k

Child 2

J_{12}^k	J_{21}^k	J_{32}^k	…	…	…	J_{r1}^k
σ_{12}^k	σ_{21}^k	σ_{32}^k	…	…	…	σ_{r1}^k

Fig. 3. Cross operator.

2.3.4. Selection

The crossing and mutation operations are carried out until λ individuals are obtained. The population made up of μ individuals and by the new λ individuals is evaluated, as explained in the following section.

The μ individuals with least cost are selected from the population made up of $\mu + \lambda$ individuals (elitist selection).

2.3.5. Evaluation

Once we have a population of individuals it is necessary to identify the best ones amongst them. To achieve that costs should be assigned to each of them.

Substituting the codified J of an individual of the population in (5), the problem becomes a linear problem, which we can solve through linear programming. Solving the problem would allow us to know the value of the objective function, ψ. The value this function takes with each individual is the cost associated to it.

Besides assigning cost to each individual, the solution of the linear problem also determines the value of the Ks belonging to the Js codified in each individual.

With the J of each individual, the non-linear problem in (5) becomes the linear problem in (10), since $A_{(i,j)}^{f_j}$, $B_{(i,j)}^{f_j}$ and $H_{(i,j)}^{f_j}$ are actual constants.

$$\Psi = \min\left[\sum_{j=1}^{n}\sum_{f_j}^{n_{ccj}} H_j^{f_j} \cdot K_j\right]$$

sujeto a :

$$\begin{cases} A_{(i,j)}^{f_j} K_i - B_{(i,j)}^{f_j} K_j \geqslant r'_{(i,j)} & \forall (i,j) \in R, \quad \forall f_j \\ K_{imin} \leqslant K_i \leqslant K_{imax} & \forall i \\ K_{jmin} \leqslant K_j \leqslant K_{jmax} & \forall i \end{cases} \tag{10}$$

where

$$A_{(i,j)}^{f_j} = \frac{a \cdot (J_i)^b}{(I_{ij1}^{f_j})^b - (J_i)^b} \quad B_{(i,j)}^{f_j} = \frac{a \cdot (J_j)^b}{(I_{jj1}^{f_j})^b - (J_j)^b}$$
$$H_j^{f_j} = \frac{a \cdot (J_j)^b}{(I_{jj1}^{f_j})^b - (J_j)^b} \tag{11}$$

The solution obtained in solving problem (10) makes all relays of the network act in a coordinated manner and in the least time possible, for the Js of each individual. The cost assigned to each individual is given by:

$$Cost = \psi \tag{12}$$

But it is possible that problem (10) may not have a solution, in which case the individual cannot be evaluated. To have a solution and a cost associated to each individual we solve the problem by eliminating restrictions. This way, a partial solution can be found, which, though it may not be the one that makes all relays act in a coordinated manner, it will at least allow a part of them to do it.

To determine the restrictions that must be eliminated, we consider the necessary condition that the relays of a mesh must fulfil in order to be **totally coordinable,** such as demonstrated in Appendixes:

$$\prod_{(i,j)\in L}\left(\frac{(I_{jj1}^{f_j})^b - (J_j)^b}{(I_{ij1}^{f_j})^b - (J_i)^b}\right) > 1 \quad \forall f_j \tag{13}$$

The algorithm used for eliminating restrictions is as follows:

1. The pair of relays *(i,j)* and fault f_j are selected, giving rise to the smallest term of (13)

2. Restriction (14), which corresponds to the (i,j) pair and to the f_j fault, is eliminated.

$$t_{ij1}^{f_j} - t_{jj1}^{f_j} \geqslant r'_{(i,j)} \tag{14}$$

3. If there is no solution, it is repeated from point 1

4. The cost is determined:

$$Cost = \psi + k \cdot n_{re} \tag{15}$$

When we eliminate restrictions, the cost associated with the individual is the value of the objective function (ψ) plus one penalization. Said penalization is a function of the number of restrictions eliminated, n_{re}. With k being a sufficiently large number, so that an individual that does fulfil one of the restrictions may have a much larger cost than any other that does fulfil them. That way it is guaranteed that an individual that fulfils all restrictions has a much lower cost than another that does not fulfil them, and, therefore, more possibilities of being selected.

2.3.6. Stopping criterion

The evolutionary algorithm stops when convergence and uniformity of the population occurs.

3. Result

Below is a reproduction of a network fed by two generators and with six lines.

The characteristics of the lines and of the generators of the network of Fig. 4 are those indicated in Tables 1 and 2.

We consider the intensity of the load assigned to each one of the relays as the highest of the intensities felt by each one of them by carrying out the contingency analysis ($N-1$). A value equal to 10.0 A is assigned to relays that have no intensity in any of the ($N-1$) configurations. The maximum load intensities considered are set forth in Table 3.

In the example, we determined the relay adjustments considering them with an SI characteristic, since that is the usual situation in the networks within our geographic area. The values of a and b for said characteristic are 0.14 and 0.02, respectively. The values that K may take depend on the relay manufacturer. In the network used as example, we have used Alstom MiCom P141 relays. The lower limit of K, K_{min} for this relay is 0.05 and its upper limit, $K_{max,}$ is 10.00. The time delay, $r'_{(i,j)}$, that we assign to all backup relays is 0.3 s.

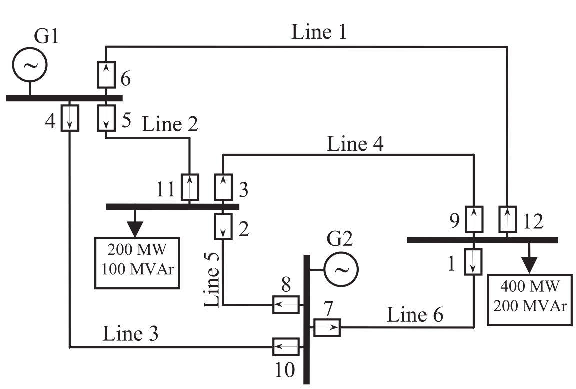

Fig. 4. Network.

Table 1
Characteristics of the generators.

	S_n (MVA)	U_n (kV)	Group	R_1, R_2, R_3 (p.u.)	X_1, X_2 (p.u.)	X_0 (p.u.)	Z_r (Ω)	Z_x (Ω)
G1	400	220	YN	0.0	1.0	3.0	0.0	0.0
G2	400	220	YN	0.0	1.0	3.0	0.0	0.0

Table 2
Characteristics of the lines.

	U_n (kV)	Long (km)	R_1 (Ω/km)	X_1 (Ω/km)	B_1, B_0 (S/km)	R_0 (Ω/km)	X_0 (Ω/km)
Line 1	220	21.90	0.084	0.426	0.0	0.326	1.299
Line 2	220	4.50	0.035	0.311	0.0	0.260	1.112
Line 3	220	24.40	0.084	0.426	0.0	0.326	1.299
Line 4	220	22.00	0.035	0.311	0.0	0.260	1.112
Line 5	220	9.40	0.084	0.426	0.0	0.326	1.299
Line 6	220	10.80	0.072	0.398	0.0	0.360	1.150

Table 3
Relay load intensity.

Relay	Load intensity in A	Relay	Load intensity in A
1	10.000	7	277.176
2	10.000	8	307.137
3	128.648	9	76.790
4	85.636	10	99.743
5	342.921	11	10.000
6	167.232	12	10.000

Once the relevant program is executed, the regulation of the overcurrent relays of the network is in Table 4. With said adjustments, the sum of the main relay operation times (cost) for triphasic (Tri) and biphasic (Bi) faults in the 0%, 50% and 100% of the length of the six lines is 47.14 s.

Figs. 5 and 6 show the maximum, average and minimum values of Cost and of Sigma, σ, of the algorithm.

In the Table 5 are the operation times, t_p and $t_{b,}$ of the main and backup relays, r_p and r_b, respectively, for triphasic and biphasic faults placed at 0%, 50% and 100% of the line, with regards to the main relay. Thus, for example, when a biphasic fault occurs at 0% of relay 5, said relay operates in 0.81 s and its backup relays 10 and 12 do it in 1.21 s and 1.11 s, respectively. For relay 11, opposite 5, said fault is in 100%, and its operation time is 0.68 s, and the operation times for its backup relays 8 and 9 are 2.15 s and 4.63 s, respectively.

The reason for the lack of operation of some backup relays ($t=\infty$) is that the intensity circulating through them (I) is null or less than their adjustment intensity (J).

Table 4
Relay adjustment parameters.

Relay	Adjustment parameters		Relay	Adjustment parameters	
	K	J		K	J
1	0.3213	12.6000	7	0.1587	277.1771
2	0.2751	57.2000	8	0.0959	307.1379
3	0.1888	128.6496	9	0.1466	76.7906
4	0.1441	85.6379	10	0.1450	99.7439
5	0.1546	342.9225	11	0.2575	38.2000
6	0.1209	167.2336	12	0.4353	10.0001
Cost	47.71 s				

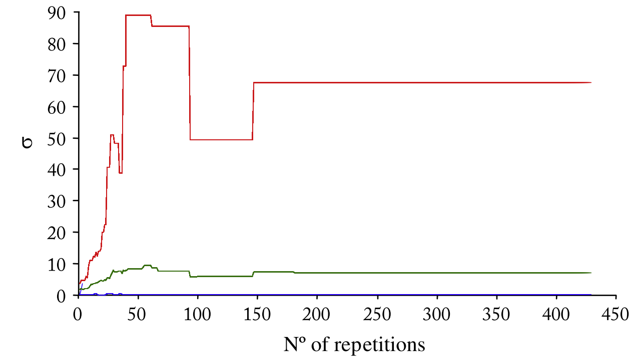

Fig. 5. Evolution of Sigma with number of repetitions.

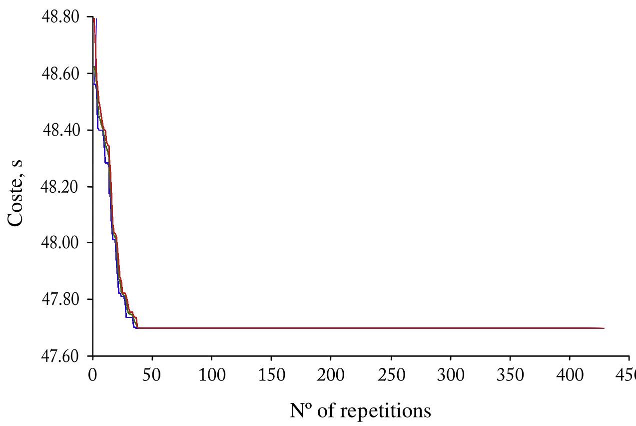

Fig. 6. Evolution of cost with number of repetitions.

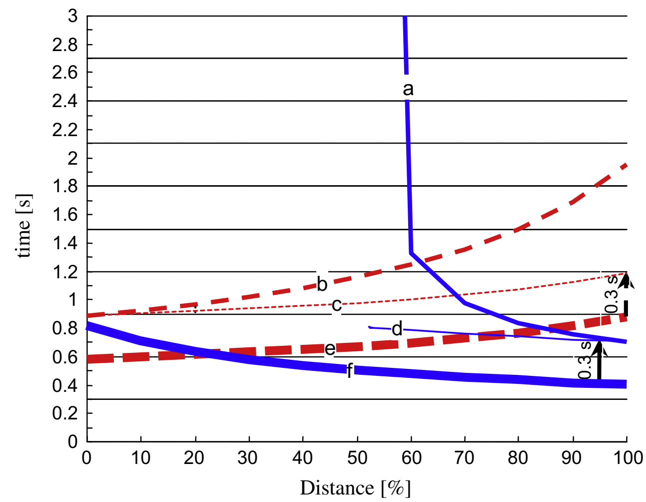

Fig. 7. Operation curves of line 5 main and backup relays.

Fig. 7, as an example, reproduces the operation curves of the main and *backup* relays of line 5, with the parameters obtained through the proposed method and without considering the intensity variations produced when the switches associated to the relays are opened. The *e* curve is that of relay 2, *f* that of relay 8, *b* that of *backup* relay of relay 2 which operates first, *a* that of *backup* relay of 8 which operates first and the *c* and *d* curves are the *e* and *f* curves, respectively, with a time delay of 0.3 s. It can be observed that the *backup* relays always operate with a time delay equal to or exceeding 0.3 s, which is the time that we have assigned to $r'_{(i,j)}$.

4. Conclusion

A new method of optimization is presented to determine the time multiplier setting and the current setting of directional overcurrent relays of an electrical network. Due to the fact the operation time of the relays of each (i,j) pair only needs the verification of a small number of restrictions, the main advantages of the method proposed above is that the evolutionary algorithm and that of the linear programming are fast. Besides, unlike the proposals carried out by authors included in the bibliography, it allows the elimination of restrictions so as to seek a solution to achieve the partial coordination of the relays.

Table 5
Operation times.

		Biphasic fault position						Triphasic fault position					
		0%		50%		100%		0%		50%		100%	
r_p	r_b	t_p	t_b	t_p	t_b	t_p	t_b	t_p	t_b	t_p	t_b	t_p	t_b
		0.50	0.97	0.57	1.58	0.74	∞	0.49	0.87	0.54	1.34	0.70	∞
	6		0.91		1.60		∞		0.79		1.26		∞
2	5	0.61	1.01	0.71	1.37	0.95	2.65	0.58	0.88	0.67	1.16	0.88	1.96
	9		1.01		∞		∞		0.88		∞		∞
3	5	0.51	1.01	0.63	1.52	0.97	4.95	0.49	0.88	0.59	1.26	0.87	2.98
	8		1.04		∞		∞		0.85		3.77		∞
4	11	0.34	0.68	0.42	∞	1.01	∞	0.32	0.64	0.40	∞	0.88	∞
	12		1.11		∞		∞		1.05		∞		∞
5	10	0.81	1.21	0.89	2.12	1.05	∞	0.73	1.03	0.79	1.62	0.88	14.1
	12		1.11		1.49		∞		1.05		1.38		∞
6	10	0.36	1.21	0.46	∞	0.91	∞	0.34	1.03	0.43	9.99	0.78	∞
	11		0.68		∞		∞		0.64		12.5		∞
7	2	0.63	0.95	0.75	1.73	0.99	∞	0.58	0.88	0.68	1.53	0.87	∞
	4		1.01		2.50		∞		0.88		1.84		∞
8	1	0.44	0.74	0.58	∞	1.04	∞	0.40	0.71	0.51	15.2	0.85	∞
	4		1.01		∞		∞		0.88		∞		∞
9	6	0.36	0.91	0.45	4.01	1.07	∞	0.34	0.79	0.42	2.38	0.88	∞
	7		0.99		1.74		∞		0.88		1.42		∞
10	1	0.36	0.74	0.46	∞	1.20	∞	0.34	0.71	0.43	∞	1.02	∞
	2		0.95		∞		∞		0.88		∞		∞
11	8	0.58	1.04	0.62	1.37	0.68	2.15	0.55	0.85	0.59	1.06	0.64	1.47
	9		1.01		1.48		4.63		0.88		1.22		2.80
12	3	0.59	0.97	0.68	3.06	1.10	∞	0.58	0.88	0.66	2.29	1.05	∞
	7		0.99		1.70		∞		0.88		1.39		∞

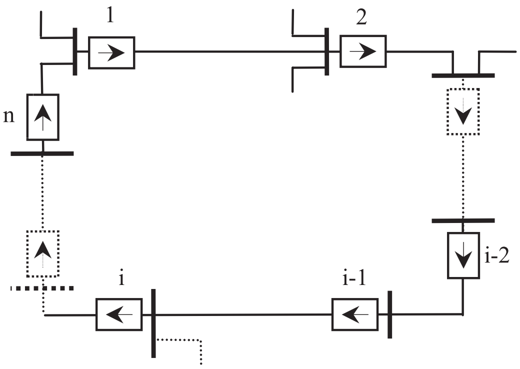

Fig. 8. Network mesh.

Appendix A

To identify the restrictions that must be eliminated, so that they coordinate the highest number of relays possible, the condition to be fulfilled by the relays belonging to the same mesh of the network to be coordinated is determined.

Beginning from the first restriction of (5), which may be put as the following:

$$\frac{a(J_i)^b}{(I_{ij1}^{f_j})^b - (J_i)^b} K_i - \frac{a(J_j)^b}{(I_{jj1}^{f_j})^b - (J_j)^b} K_j \geqslant r'_{(i,j)} \quad \forall (i,j) \in R, \quad \forall f_j \tag{16}$$

Operating, it is obtained that:

$$K_j \leqslant F_{(i,j)}^{f_j} K_i + G_{(i,j)}^{f_j} r'_{(i,j)} \quad \forall (i,j) \in R, \quad \forall f_j \tag{17}$$

where

$$F_{(i,j)}^{f_j} = \left(\frac{(I_{jj1}^{f_j})^b - (J_j)^b}{(I_{ij1}^{f_j})^b - (J_i)^b} \right) \left(\frac{J_i}{J_j} \right)^b \qquad G_{(i,j)}^{f_j} = \frac{1}{a} \left(1 - \frac{(I_{jj1}^{f_j})^b}{(J_j)^b} \right) \tag{18}$$

Eq. (17) represents the relation that K_i and K_j of each pair of relays (i,j) must comply with in order to coordinate, when an f_j fault is produced.

In order to determine the condition that relays of a mesh to be coordinated need to fulfil, the study will focus on the mesh in Fig. 8, in which, in order to facilitate the exposition, only the unidirectional relays have been represented.

The system of equations resulting from the application of (17) to each pair of relays of the mesh in the figure is:

$$\begin{cases} K_1 \leq F_{(n,1)}^{f_1} K_n + G_{(n,1)}^{f_1} r'_{(n,1)} & \forall f_1 \\ K_2 \leq F_{(1,2)}^{f_2} K_1 + G_{(1,2)}^{f_2} r'_{(1,2)} & \forall f_2 \\ \dots\dots\dots & \\ K_{i-2} \leq F_{(i-3,i-2)}^{f_{i-2}} K_{i-3} + G_{(i-3,i-2)}^{f_{i-2}} r'_{(i-3,i-2)} & \forall f_{i-2} \\ K_{i-1} \leq F_{(i-2,i-1)}^{f_{i-1}} K_{i-2} + G_{(i-2,i-1)}^{f_{i-1}} r'_{(i-2,i-1)} & \forall f_{i-1} \\ K_i \leq F_{(i-1,i)}^{f_i} K_{i-1} + G_{(i-1,i)}^{f_i} r'_{(i-1,i)} & \forall f_i \\ K_{i+1} \leq F_{(i,i+1)}^{f_{i+1}} K_i + G_{(i,i+1)}^{f_{i+1}} r'_{(i,i+1)} & \forall f_{i+1} \\ \dots\dots\dots & \\ K_n \leq F_{(n-1,n)}^{f_n} K_{n-1} + G_{(n-1,n)}^{f_n} r'_{(n-1,n)} & \forall f_n \end{cases} \tag{19}$$

The term $A_{(i-2,i-1)}^{f}$ represents the coefficient A of relay pair $(i-2, i-1)$ where relay $i-2$ is the relay found two relays behind relay i and the $i-1$ relay is located one relay behind relay i. Thus, for example, if $i = 2$ the previous relay pair would represent the $(n,1)$ pair. To refer to a relay situated before relay i the plus sign (+) is used. So relay pair $(i,i+1)$, when $i = n$, corresponds to the pair $(n,1)$.

In previous expressions, L is the group of relay pairs (*backup*, main) belonging to one mesh.

The determination of the K of any given relay can be carried out through the system of Eqs. (19). Therefore, K of relay i, K_i, would be:

$$K_i \leqslant \frac{\sum_{p=1}^{n-1} \left[\left(\prod_{q=1}^{n-p} F_{(i-q,i-q+1)}^{f_{i-q+1}} \right) G_{(i+p-1,i+p)}^{f_{i+p}} r'_{(i+p-1,i+p)} \right]}{1 - \prod_{(i,j)\in L} F_{(i,j)}^{f_j}} + \frac{G_{(i-1,i)}^{f_i} r'_{(i-1,i)}}{1 - \prod_{(i,j)\in L} F_{(i,j)}^{f_j}} \quad \forall f_i, \forall f_{i-1}, \dots, \forall f_{i-n+1}, \forall f_{i-n+2}, \dots, \forall f_{i-n+n} \tag{20}$$

Expression (20) represents the relation that must be verified by the K of each relay of the mesh to be coordinated.

From the adjustment limits of J given in (4), for relay j, it is verified that:

$$I_{jj1}^{f_j} \geqslant J_j \quad \forall f_j \tag{21}$$

And in relay i that:

$$I_{ij1}^{f_j} \geqslant J_i \quad \forall f_j \tag{22}$$

Considering the relations of (21) and (22), it can be concluded that, in (18), the following are fulfilled:

$$F_{(i,j)}^{f_j} \geqslant 0 \quad G_{(i,j)}^{f_j} \leqslant 0 \tag{23}$$

The value of $r'_{(i,j)}$, depends on the relay technology, and it is always more than zero.

$$r_{(i,j)'} > 0 \quad \forall f_j \tag{24}$$

Since the value that K may take is always positive, considering (23) and (24), the following must be verified:

$$\prod_{(i,j)\in L} F_{(i,j)}^{f_j} > 1 \quad \forall f_j \tag{25}$$

Substituting (18) in (25), the latter can be expressed in the following manner:

$$\prod_{(i,j)\in L} F_{(i,j)}^{f_j} = \prod_{(i,j)\in L} \left(\frac{J_i}{J_j} \right)^b \prod_{(i,j)\in L} \left(\frac{(I_{jj1}^{f_j})^b - (J_j)^b}{(I_{ij1}^{f_j})^b - (J_i)^b} \right) \quad \forall f_j \tag{26}$$

The first factor of (28), applied to the mesh of Fig. 8, is:

$$\prod_{(i,j)\in L} \left(\frac{J_i}{J_j} \right)^b = 1 \tag{27}$$

In order for the inequality (27) to be fulfilled, it must be therefore be verified that:

$$\prod_{(i,j)\in L} \left(\frac{(I_{jj1}^{f_j})^b - (J_j)^b}{(I_{ij1}^{f_j})^b - (J_i)^b} \right) > 1 \quad \forall f_j \tag{28}$$

Inequality (28) is the necessary condition that the relays of a mesh must fulfil in order to be completely coordinable.

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