A Linear Dynamic Model for Asynchronous Wind Turbines With Mechanical Fluctuations

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***Abstract*—In this paper, a specific linear dynamic model for an asynchronous machine is presented. This model is based on the balanced dynamic model of the asynchronous machine and it is developed for asynchronous wind turbines when the mechanical power (wind power) has sinusoidal fluctuations. The dynamic and the proposed linear models for real and reactive powers and voltage fluctuations analysis are considered and compared.**

***Index Terms*—Asynchronous rotating machine, power quality, wind energy.**

I. Introduction

WIND energy has become an increasing source of electrical energy production in recent years. For this reason, it is necessary to study the possible impact a wind turbine will produce on the power network where it is connected. In order to investigate the effects, suitable wind energy models must be used.

In general, the effects of asynchronous wind turbines on power systems can be classified in two groups: 1) steady-state security and 2) power quality. The goal of steady-state security is to seek network power stability conditions when the wind power is injected, whereas, power quality analysis studies the effects of wind power fluctuations in the form and level of electrical waves (harmonics, fluctuations, etc.).

In order to assess the impact of wind energy on the steady-state security of power networks, the methodology is based on load flow analysis, and asynchronous wind turbines can be modeled by PQ or RX (impedance) nodal elements where the mechanical power (produced by the wind) is considered from a probabilistic point of view [1], [2]. When the study of power quality is required, in general, the power network from the wind turbine is modeled by a Thevenin equivalent circuit, and the asynchronous generator can be presented by a PQ model or a dynamic one, which is a third-order induction machine.

The PQ model is commonly used for analysis of stationary voltage variations (flicker) [3]–[4], and its main advantage is its simplicity.

The third-order dynamic model [5] is used by several authors when connection processes or exhaustive analyses are needed [5]–[7]. The advantage of the dynamic model is the accuracy of results, but its complexity is not appropriate for large studies.

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In this paper, a linear dynamic model is proposed. This model is based on the dynamic linear equations [5], [8] considering the machine at the operating point (defined by the steady-state model) and sinusoidal fluctuations of mechanical power. The resulting model, after applying a Fourier transform, results in a complex matrix equation such as $\Delta \underline{X} = M \Delta \underline{b}$, where the complex 3×1 matrix $\Delta \underline{\mathbf{X}}$ represents sinusoidal variations of internal voltages and slip, $\underline{\mathbf{M}}$ is a constant complex 3×3 matrix and $\Delta \underline{\mathbf{b}}$ is a 3×1 matrix with one nonzero component which represents the mechanical power sinusoidal fluctuation. Using the proposed linear model, other electrical variables (real and reactive power and output voltage) of the asynchronous wind turbine are defined. A comparative study of the dynamic and the proposed linear models are shown.

The proposed linear model of an asynchronous wind turbine can be easily implemented in power system analysis.

II. Model of Mechanical Power in the Wind Turbine

The model of mechanical power can be expressed by the following terms:

$$P_m(t) = P_{m,0} + \Delta P_m \tag{1}$$

where $P_{m,0}$ is the 10-min mean value of the power of the wind turbine, obtained from the wind, which is defined by a Weibull or a Rayleigh distribution [2], [9].

ΔP_m is the power caused by tower shadow, wind shear, and rotational sampling. The wind power fluctuations are considered to have a value proportional to $P_{m,0}$. The frequency of ΔP_m is defined by the speed of the blades, and this is a function of synchronous speed Ω_s and slip. Measurements taken by the authors have shown this to be an accurate way to model tower shadow effect, as can be seen in Fig. 1(a) and (b) which represent, respectively, the electrical power harmonic spectrum and real/reactive powers ratio of a wind turbine (for the sake of simplicity, data are given in Appendix II). At the moment of the measurements, the wind turbine was injecting 102 kW to the electrical power network. In Fig. 1(a), the fundamental is excluded.

Another mechanical power term P_h can be considered. This is the power from wind turbulence defined by a Normal distribution, which is similar to a stochastic noise. This term is not taken into account in this paper.

In this paper, the following hypotheses are considered.

1) The $P_{m,0}$ component of mechanical power is constant
2) The ΔP_m mechanical power can be expressed as

$$\Delta P_m(t) = P_s \sin(\theta(t)) \tag{2}$$

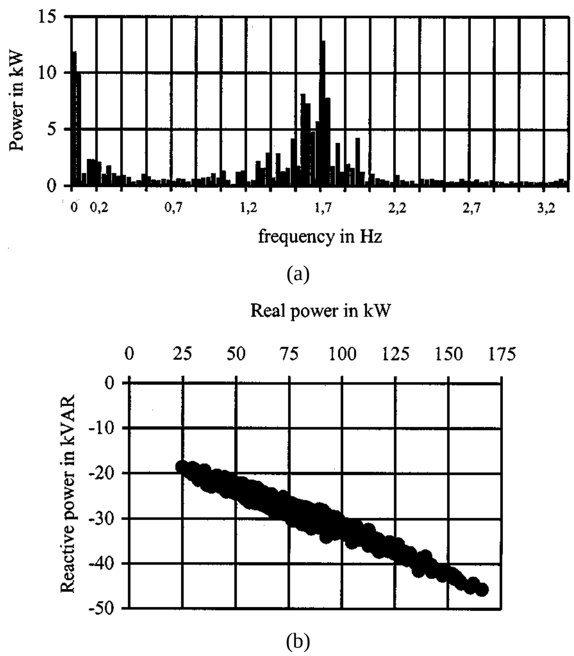


Fig. 1. (a) Power harmonic spectrum. (b) Real/reactive power ratios.

where P_s is the amplitude of the mechanical sinusoidal fluctuations and $\theta(t)$ is the mechanical angle of the turbine, defined by means of the following equations:

$$(1-s)\Omega_s = \frac{d\theta(t)}{dt} \Rightarrow \theta(t) = \theta(t_0) + \int_{t_0}^{t} (1-s)\Omega_s \, dt \quad (3)$$

where

- s slip of the induction generator;
- $\Omega_s = 3 \cdot 2\omega_s/(rp)$ speed corresponding to the tower shadow effect in rad/s, where $2\omega_s/(rp)$ is the wind turbine synchronous speed;
- ω_s synchronous speed in rad/s of the induction generator ($2\pi 50$ in Europe);
- r gear box ratio, which is in this case 1 : 44.38;
- p number of poles, four in this case.

III. Induction Generator Models

A. Dynamic Model

The induction generator can be modeled as a Thevenin equivalent voltage source $\underline{E}'$ behind the impedance $R_s + jX'$, as can be seen in Fig. 2 [5], where it is represented in front of a network and a compensator bank. This dynamic model is defined by considering balanced work and neglecting the electromagnetic dynamic effects of the stator, and it is known as third-order induction machine model.

The value of $\underline{E}'$ can be calculated by integrating the following:

$$\frac{d\underline{E}'}{dt} = -j\omega_s s \underline{E}' - \frac{1}{T_0'} \left(\underline{E}' - j(X_0 - X')\underline{I}_s\right) \quad (4)$$

where T_0', X' and X_0 are the following constants:

$$T_0' = \frac{X_r + X_m}{2\pi f_s R_R} \quad X_0 = X_r + X_m \quad X' = X_s + \frac{X_r X_m}{X_r + X_m}$$

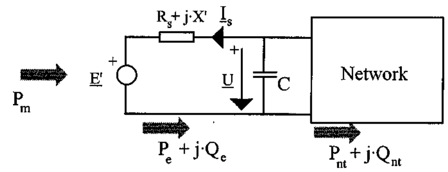


Fig. 2. Dynamic model of an asynchronous generator.

and

- X_r rotor leakage reactance;
- X_s stator leakage reactance;
- X_m magnetizing reactance;
- R_s stator resistance;
- R_r rotor resistance;
- f_s network frequency;
- $\underline{I}_s$ stator current.

The electrical equation of the stator current is

$$\underline{I}_s = \frac{\underline{U} - \underline{E}'}{R_s + jX'} \quad (5)$$

where $\underline{U}$ is the stator voltage.

The electromechanical equation is

$$-\frac{P_m}{1-s} + P_e' = -2H\frac{ds}{dt} \Rightarrow P_m - P_e = 2H(1-s)\frac{ds}{dt} \quad (6)$$

where

- P_m mechanical power ($P_m < 0$ for generation);
- H inertia constant;
- P_e real power.

The electrical power P_e is calculated by

$$P_e = P_e'(1-s) = \mathrm{Re}\left\{\underline{E}'\underline{I}_s*\right\}. \quad (7)$$

In [6], more general expressions for (4)–(7) are presented, where the mechanical power is defined by (1) and (2), where it is shown that stator currents of an asynchronous generator can be expressed by Fourier series, and wind turbine and power network interaction is analyzed using an iterative process. As given in Appendix I of this paper, simplified expressions of general equations in [6] are shown, when the sinusoidal amplitude Ω (modulation wave) and the sinusoidal oscillation of electrical variables ω (carried wave) fulfill $\Omega \ll \omega$ as happens in the wind turbine. Consequently, (4)–(7) can be directly calculated without needing Fourier decomposition and Bessel functions.

B. Dynamic Simulation of the Wind Turbine With Mechanical Fluctuations

When a wind turbine, connected to a power system modeled by a Thevenin circuit, such as in Fig. 3, has mechanical fluctuations ΔP_m, the electrical variables $\underline{E}'$, $\underline{I}_s$, P_e, Q_e are also fluctuating.

The dynamic asynchronous generator behavior with mechanical power fluctuations depends on several input variables, such as amplitude of the mechanical fluctuation P_s, frequency of the mechanical fluctuations Ω, short-circuit power S_{cc}, and X/R ratio of S_{cc}.

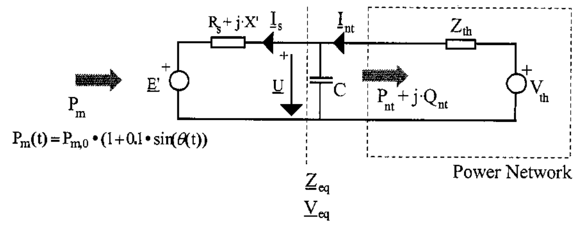


Fig. 3. Asynchronous wind energy converter.

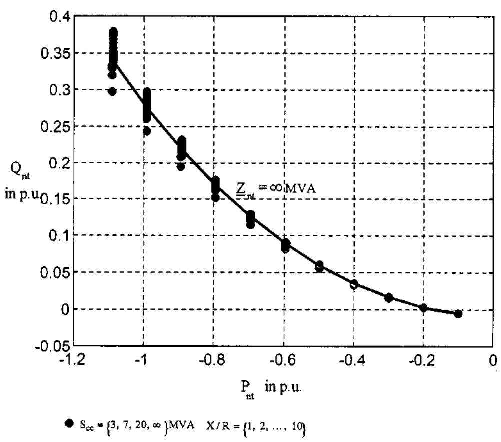


Fig. 4. Variables P_{nt} and Q_{nt} under several steady-state situations.

C. PQ Model

The PQ model of the asynchronous generator is presented in [1]–[4]. This model defines the asynchronous generator as a power source where the real power P_{nt} is obtained by measurements or practical considerations and the reactive power Q_{nt} is calculated by the expression: $Q_{nt} = q_0 + q_1 P_m + q_2 P_m^2$, where q_0, q_1 and q_2 are constants, that depend on the type of generator. This model is very easy to implement and practical but is not accurate.

Fig. 4 shows variables P_{nt} and Q_{nt} for the asynchronous generator of Appendix II in several steady-state situations defined by: $P_m = \{0.1, 0.2, \ldots, 1, 1\}$, $S_{cc} = \{3, 7, 20, \infty\}$ MVA, and $X/R = \{1, 2, \ldots, 10\}$. In Fig. 4, the power values for $S_{cc} = \infty$ are represented by a solid line, and the other cases are represented as solid circles. The results of Fig. 4 show that the PQ model for steady-state situations must be defined for a specific short-circuit impedance Z_{nt} and $P_{nt} \approx P_m$.

Fig. 5 shows the variables P_{nt} and Q_{nt} for the asynchronous generator of Appendix II under several dynamic situations [sinusoidal fluctuations of mechanical power in (2)] defined by: $P_m = \{0.4, 0.6, 0.8, 1.0\}$, $P_s = 0.1 P_m$, $S_{cc} = 7$ MVA, and $X/R = 5$. In Fig. 5, the steady-state power values are represented by the decreasing curve and the fluctuations as circular curves. The results of Fig. 5 show that for sinusoidal mechanical power situations the power fluctuations ΔP_m and ΔP_{nt} have different moduli, different angles, and their ratio is different from that obtained in steady-state situations.

IV. Proposed Linear Dynamic Model

If the induction generator is assumed to be in an initial steady-state situation, defined by the values $\underline{E}'_0$, s_0, $\underline{I}_0$, P_0, $\underline{U}_0$ and

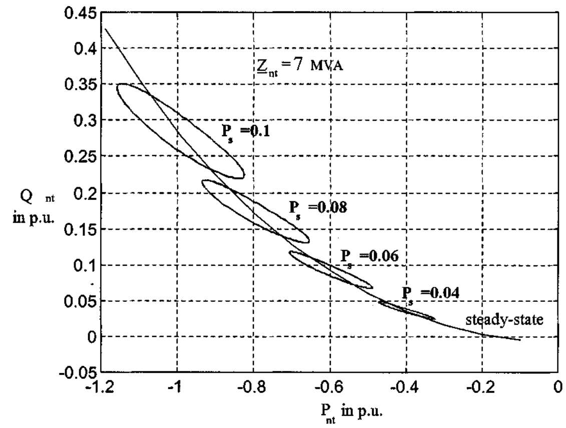


Fig. 5. Variables P_{nt} and Q_{nt} obtained under several dynamic situations.

considering a circuit such as that in Fig. 3, (4) can be expressed as

$$\frac{d\left(\underline{E}'_0 + \Delta\underline{E}'\right)}{dt} = -\left(\underline{E}'_0 + \Delta\underline{E}'\right)\left(j\omega_s(s_0 + \Delta s) + \underline{z}\right) + \underline{z}'\underline{U}_{eq} \quad (8)$$

where

$$\underline{z}' = \frac{1}{T'_0}\frac{j(X_0 - X')}{R_s + jX' + \underline{Z}_{eq}}$$

$$\underline{z} = \frac{1}{T'_0}\left(1 + \frac{j(X_0 - X')}{R_s + jX' + \underline{Z}_{eq}}\right).$$

Assuming small changes [8] $\Delta E' \Delta s = 0$, (8) can be written as

$$\frac{d(\Delta E_r)}{dt} = -\alpha\Delta E_r + (\omega_s s_0 + \beta)\Delta E_m + \omega_s E'_{m0}\Delta s$$

$$\frac{d(\Delta E_m)}{dt} = -(\omega_s s_0 + \beta)\Delta E_r - \alpha\Delta E_m - \omega_s E'_{r0}\Delta s \quad (9)$$

where

$$\underline{E}'_0 = E'_{r,0} + jE'_{m,0} \quad \Delta\underline{E}' = \Delta E'_r + j\Delta E'_m$$

$$\alpha + j\beta = \underline{z} = \frac{1}{T'_0}\left(1 + \frac{j(X_0 - X')}{R_s + jX' + \underline{Z}_{eq}}\right).$$

The electrical power P_e can be calculated, considering small changes [8] $\Delta E' \Delta E'^* \approx 0$ as

$$P_{e,0} + \Delta P_e = \mathrm{Re}\left\{\underline{E}'\underline{I}_s^*\right\} \approx P_{e,0} + c_e\Delta E'_r + d_e\Delta E'_m \quad (10)$$

where

$$c_e = -G + \mathrm{Re}\left\{\underline{I}_{s,0}\right\} \quad d_e = -B + \mathrm{Im}\left\{\underline{I}_{s,0}\right\}$$

$$G + jB = \frac{\underline{E}'_0}{R_s - jX' + \underline{Z}^*_{eq}}.$$

For the mechanical power fluctuations ΔP_m defined in (2), the angle $\theta(t)$ in (3) can be expressed as

$$\theta(t) = \theta_0 + \int_0^t (1 - s)\Omega_s dt = \theta_0 + \int_0^t (1 - s_0 - \Delta s)\Omega_s dt$$

$$= \theta_0 + \Omega_0 t - \int_0^t \Delta s \Omega_s dt = \theta_0 + \Omega_0 t + \Delta\theta$$

where $\Omega_0 = (1 - s_0)\Omega$ and θ_0 is an initial value for the angle.

If we assume $\theta_0 = 0$ and $\sin(\Delta\theta) = \Delta\theta \approx 0$, the result is

$$\Delta P_m(t) = P_s \sin(\Omega_0 t). \quad (11)$$

Taking (6) and (9)–(11) into account, the asynchronous machine equations can be expressed as follows:

$$\frac{d}{dt}\begin{pmatrix} \Delta E'_r \\ \Delta E'_m \\ \Delta s \end{pmatrix} = \begin{pmatrix} -\alpha & \omega_s s_0 + \beta & \omega_s E'_{m0} \\ -\omega_s s_0 - \beta & -\alpha & -\omega_s E'_{r0} \\ \dfrac{-c_e}{h} & \dfrac{-d_e}{h} & 0 \end{pmatrix} \cdot \begin{pmatrix} \Delta E'_r \\ \Delta E'_m \\ \Delta s \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dfrac{1}{h} \end{pmatrix} \Delta P_m \quad (12)$$

where $h = 2H(1 - s_0)$.

The previous equations system can be represented by the linear first-order differential equation

$$\dot{x} = A'x + b\Delta P'_m. \quad (13)$$

In order to obtain the oscillation modes of the system, we can examine the eigenvalues of the characteristic matrix A' or the roots of the characteristic equation, by solving $\det[\lambda\mathbf{I} - \mathbf{A}] = 0$, where $\mathbf{1}$ is the unitary matrix.

As an example, for the asynchronous generator of Appendix II, with $P_s = 0.1$ p.u., $P_m = 1$ p.u., $S_{cc} = 7$ MVA, and $X/R = 5$, the following roots are obtained: $(-3.6604 + 12.0040j)$, $(-3.6604 - 12.0040j)$, and (-5.5867). On the other hand, the rotational speed of mechanical power $\Delta P'_m$ is $\Omega_0 = 10.6916$ rad/s, which is near to the frequency oscillations of 12.0040. Therefore, the electrical variables of the generator will show higher amplitude fluctuations for rotational speed Ω_0, and consequently the steady-state models (PQ, RX) are not suitable when mechanical power fluctuations exist.

In the steady-state situation of (13), from sinusoidal fluctuations of $\Delta P'_m$, the Fourier transform can be applied, giving

$$X(\omega) = (j\omega 1 - A')^{-1} b \Delta P'_m(\omega). \quad (14)$$

In binomial notation, (14) can be expressed as

$$\begin{pmatrix} \Delta E'_{r,r} + j\Delta E'_{r,m} \\ \Delta E'_{m,r} + j\Delta E'_{m,m} \\ \Delta s_r + j\Delta s_m \end{pmatrix} = (j\Omega_0 1 - A')^{-1} b P_s. \quad (15)$$

From (15), the sinusoidal fluctuations of variables can be defined as

$$\begin{aligned} \Delta E'_r &= \Delta E'_{r,r} \sin(\Omega_0 t) + \Delta E'_{r,m} \cos(\Omega_0 t) \\ \Delta E'_m &= \Delta E'_{m,r} \sin(\Omega_0 t) + \Delta E'_{m,m} \cos(\Omega_0 t) \\ \Delta s &= \Delta s_r \sin(\Omega_0 t) + \Delta s_m \cos(\Omega_0 t). \end{aligned} \quad (16)$$

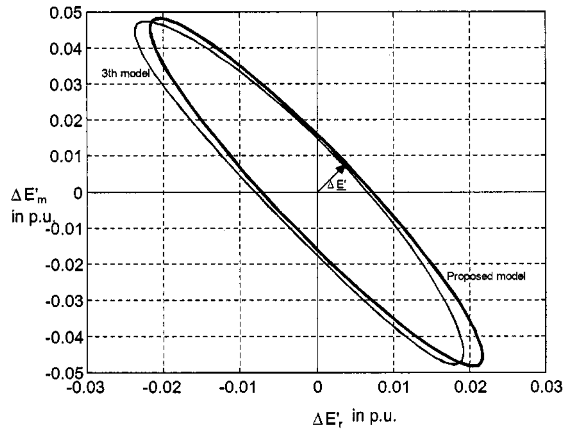


Fig. 6. Comparative results of internal voltage fluctuations between third-order dynamic model and proposed model $\underline{E}_0 = 0.8521 + j0.2489$.

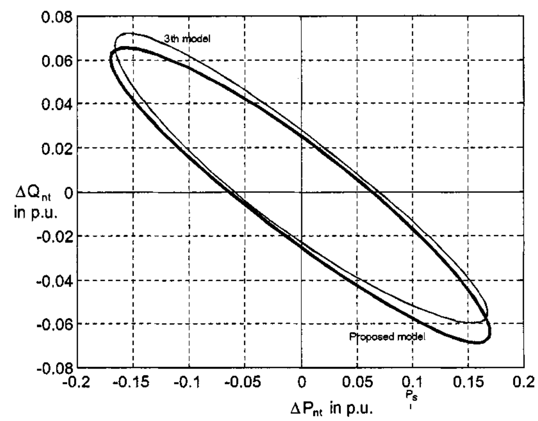


Fig. 7. Comparative results of power fluctuations between third-order dynamic model and proposed model $P_{nt,0} = -0.992$ p.u., $Q_{nt,0} = 0.2786$ p.u.

For this example, the steady-state results are

$$\begin{aligned} &\Delta E'_{r,r} = 0.0032 && \Delta E'_{m,r} = -0.0227 && \Delta s_r = 0.0023 \\ &\Delta E'_{r,m} = -0.0216 && \Delta E'_{m,m} = 0.0429 && \Delta s_m = -0.0002. \end{aligned}$$

In Figs. 6 and 7, the numerical comparative results obtained for the example are shown, between the third-order dynamic model and the proposed linear dynamic model.

V. Proposed Linear Dynamic Model Applied to a Wind Park

The proposed linear dynamic model can be applied to a wind park consisting of several asynchronous generators. In this case the variation of stator current, for machine number "i," is expressed as

$$\Delta \underline{I}_{s,i} = \frac{\Delta \underline{U}_i - \Delta \underline{E}'_i}{R_{s,i} + jX'_i} \quad (17)$$

where $\Delta \underline{U}_i$ is the nodal voltage defined by nodal analysis of the circuit in Fig. 8

$$\Delta \underline{U} = \underline{K} \Delta \underline{E}'. \quad (18)$$

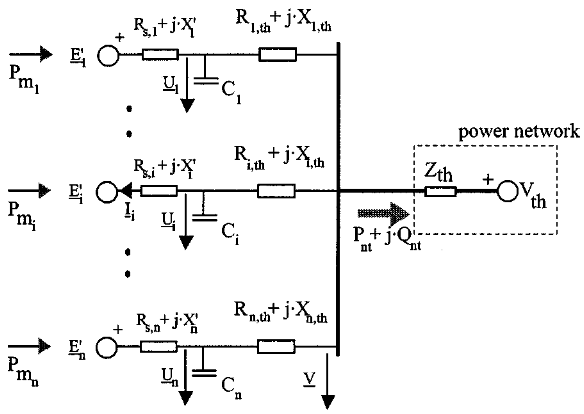


Fig. 8. Dynamic model of wind park.

Applying (4), (6), (11), and (17) to the system in Fig. 6, the result is a matrix expression such as

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} A_{1,1} & \cdots & A_{1,2} & \cdots & A_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{i,1} & \cdots & A_{i,i} & \cdots & A_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n,1} & \vdots & A_{n,i} & \cdots & A_{n,n} \end{bmatrix}\begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{bmatrix} \tag{19}$$

where

- $\mathbf{x_i}$ 3×1 vector similar to vector $\mathbf{x}$ in (12);
- $\mathbf{A_{i,i}}$ 3×3 matrix similar to matrix $\mathbf{A}$ in (12), with parameters of machine i;
- $\mathbf{A_{i,k}}$ 3×3 matrix that represents the relationship between variables of machines i and k;
- $\mathbf{b_i}$ 3×1 vector similar to vector $b\Delta P'_m$ in (12).

The values contained in the above-mentioned matrices are as follows:

$$A_{i,i} = \begin{pmatrix} -\alpha_{i,i} & \omega_s s_{0,i} + \beta_{i,i} & \omega_s E'_{m0,i} \\ -\omega_s s_{0,i} - \beta_{i,i} & -\alpha_{i,i} & -\omega_s E'_{r0,i} \\ \dfrac{-c_{i,i}}{h_i} & \dfrac{-d_{i,i}}{h_i} & 0 \end{pmatrix}$$

$$A_{i,k} = \begin{pmatrix} \alpha'_{i,k} & -\beta'_{i,k} & 0 \\ \beta'_{i,k} & \alpha'_{i,k} & 0 \\ \dfrac{-c_{i,k}}{h_i} & \dfrac{-d_{i,k}}{h_i} & 0 \end{pmatrix}$$

$$\alpha_{i,i} + j\beta_{i,i} = \frac{1}{T'_{0,i}}\left(1 + \frac{j(X_{0,i} - X'_i)\left(1 - \underline{K}_{i,i}\right)}{R_{s,i} + jX'_i}\right)$$

$$\alpha'_{i,k} + j\beta'_{i,k} = \frac{1}{T'_{0,i}}\frac{j(X_{0,i} - X'_i)\underline{K}_{i,k}}{R_{s,i} + jX'_i}$$

$$c_{i,i} = \mathbf{Re}\left\{\underline{I}_{s,0,i}\right\} + G_{i,i}$$

$$d_{i,i} = \mathbf{Im}\left\{\underline{I}_{s,0,i}\right\} + B_{i,i}$$

$$G_{i,i} + jB_{i,i} = \frac{\underline{E}_{i,0}\left(\underline{K}_{i,i} - 1\right)^*}{R_{s,i} - jX'_i}$$

$$c_{i,k} + jd_{i,k} = \frac{\underline{E}_{i,0}\left(\underline{K}_{i,k}\right)^*}{R_{s,i} - jX'_i}$$

$$h_i = 2H_i(1 - s_{i,0})$$

$\underline{K}_{i,i}$, $\underline{K}_{i,k}$ are elements of matrix $\underline{\mathbf{K}}$.

The steady-state situation of (19), for sinusoidal fluctuations of $\Delta P'_m$, can be obtained by superposition. Therefore, for $\Delta E'_r$, $\Delta E'_m$ and Δs, the following general expression can be written

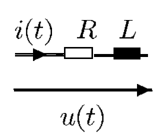
$$x(t) = \sum_{i=1}^{n} X_{r,i}\sin(\Omega_{0,i}t) + X_{m,i}\cos(\Omega_{0,i}t). \tag{20}$$

VI. Conclusions

In this paper, a linear dynamic model for an asynchronous wind turbine with mechanical sinusoidal fluctuation was developed. This proposed linear model presents good accuracy compared to the dynamic model. The proposed model has the following advantages:

1) it has low complexity;
2) it permits modal analysis, direct solution of differential equations, and easy implementation in several conventional tools for power system analysis (load flow analysis and power system stability).

Appendix I
Analysis of Linear Circuits. Electrical Variables With Amplitude Modulation



$$i(t) = I(t)\sin(\omega t) = \sqrt{2}I(1 + m\sin(\Omega t + \phi))\sin(\omega t)$$

$i(t)$ can be expressed by

$$i_0(t) = \sqrt{2}I\sin(\omega t)$$

$$i_-(t) = \frac{\sqrt{2}\mathrm{Im}}{2}\cos\left((\omega - \Omega)t - \phi\right)$$

$$i_+(t) = \frac{\sqrt{2}\mathrm{Im}}{2}\cos\left((\omega + \Omega)t + \phi\right)$$

or, in complex expressions

$$\overline{I} = I_r + jI_m$$

$$I_{0,r} = I \qquad I_{0,m} = 0$$

$$I_{-,r} = \frac{\mathrm{Im}}{2}\sin(\phi) \qquad I_{-,m} = \frac{\mathrm{Im}}{2}\cos(\phi)$$

$$I_{+,r} = -\frac{\mathrm{Im}}{2}\sin(\phi) \quad I_{+,m} = \frac{\mathrm{Im}}{2}\cos(\phi).$$

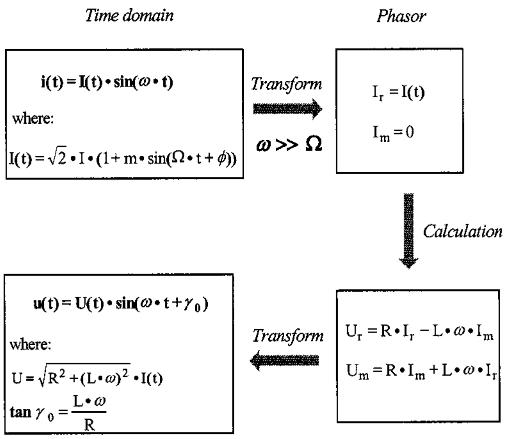


Fig. 9. Time domain and phasor analyses.

$u(t)$ in complex expressions can be seen in the following equations ($\omega \gg \Omega$):

$$
\begin{aligned}
U_{0,m} &= RI_{0,m} + L\omega I_{0,r}\\
U_{-,r} &= RI_{-,r} - L(\omega-\Omega)I_{-,m} \approx RI_{-,r} - L\omega I_{-,m}\\
U_{-,m} &= RI_{-,m} + L(\omega-\Omega)I_{-,r} \approx RI_{-,m} + L\omega I_{-,r}\\
U_{+,r} &= RI_{+,r} - L(\omega+\Omega)I_{+,m} \approx RI_{+,r} - L\omega I_{+,m}\\
U_{+,m} &= RI_{+,m} + L(\omega+\Omega)I_{+,r} \approx RI_{+,m} + L\omega I_{+,r}
\end{aligned}
$$

and $u(t)$ in a time domain expression is

$$
\begin{aligned}
u(t) = &U_{0,r}\sin(\omega t) + U_{0,m}\cos(\omega t)\\
&+ U_{-,r}\sin((\omega-\Omega)t) + U_{-,m}\cos((\omega-\Omega)t)\\
&+ U_{+,r}\sin((\omega+\Omega)t) + U_{+,m}\cos((\omega+\Omega)t)
\end{aligned}
$$

or:

$$
\begin{aligned}
u(t) = &U_0\sin(\omega t + \gamma_0) + U_-\cos((\omega-\Omega)t - \gamma_-)\\
&+ U_+\cos((\omega+\Omega)t - \gamma_+)
\end{aligned}
$$

where[1]

$$U_0 = \sqrt{(U_{0,r})^2 + (U_{0,m})^2} = \sqrt{R^2+(L\omega)^2}\, I$$

$$tg(\gamma_0) = \frac{U_{0,m}}{U_{0,r}} = \frac{L\omega}{R}$$

$$U_- = U_+ = \sqrt{(U_{-,r})^2 + (U_{-,m})^2} = \sqrt{R^2+(L\omega)^2}\,\frac{\mathrm{Im}}{2}$$

$$\gamma_- = \phi - \gamma_0 \quad \gamma_+ = -\phi - \gamma_0.$$

Therefore, $u(t)$ can be expressed by

$$u(t) \approx \left(\sqrt{R^2+(L\omega)^2}\right) I(t)\sin(\omega t + \gamma_0).$$

The summary of the last analysis can be seen in Fig. 9, which shows that the analysis of a linear circuit with amplitude modulation variables of frequency $F(\Omega = 2\pi F)$ and carried frequency $f(\omega = 2\pi f)$, can be solved in steady-state when $\Omega \ll \omega$, like a circuit of only one frequency f and electrical variable with amplitude variation in the time.

[1] $\tan(a \mp b) = \frac{\tan(a)\mp\tan(b)}{1\pm\tan(a)\tan(b)}$.

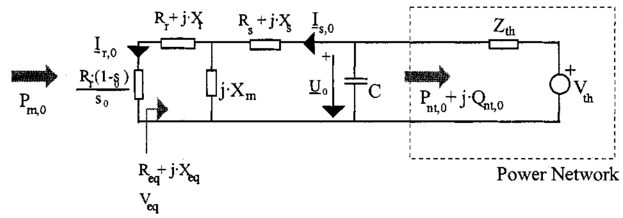


Fig. 10. RX model of the asynchronous machine in the network.

APPENDIX II
WIND TURBINE DATA

Base voltage	= 660 V.
Base power	= 350 kvar.
X_r	rotor reactance 0.0639 p.u.
X_s	stator reactance 0.187 81 p.u.
R_r	rotor resistance 0.006 12 p.u.
R_s	stator resistance 0.005 71 p.u.
X_m	magnetization reactance 2.78 p.u.
C	capacitance, $C = 1.1475 \times 10–3$ p.u.
I_s	stator current.
I_r	rotor current.
U	output voltage.
H	$= 3.025$ s.
ω_s	$= 100\pi$ rad/s.
V_{th}	$= 1$ p.u. the network Thevenin equivalent voltage.

APPENDIX III
OPERATING POINT OF AN ASYNCHRONOUS WIND TURBINE

The initial state of an asynchronous wind turbine can be defined by its RX model, shown in Fig. 10.

The mechanical power and the rotor current are defined by

$$P_{m,0} = (I_{r,0})^2\frac{R_r(1-s_0)}{s_0}$$

$$I_{r,0} = \frac{V_{eq}^2}{X_{eq}^2 + \left(R_{eq} + \frac{R_r(1-s_0)}{s_0}\right)^2}.$$

By replacing $I_{r,0}$ in the $P_{m,0}$ equation, the result is the second-order equation

$$
\begin{aligned}
&s_0^2\left(X_{eq}^2 + R_{eq}^2 - 2R_{eq}R_r + R_r^2 + \frac{V_{eq}^2R_r}{P_{m,0}}\right)\\
&\quad + s_0\left(2R_{eq}R_r - 2R_r^2 - \frac{V_{eq}^2R_r}{P_{m,0}}\right) + R_r^2 = 0.
\end{aligned}
$$

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