

# Design of Large Rural Low-Voltage Networks Using Dynamic Programming Optimization

Eloy Díaz-Dorado, Edelmiro Miguez, *Member, IEEE*, and José Cidrás, *Member, IEEE*

**Abstract**—The purpose of this paper is to establish algorithms in the design of rural low-voltage (lv) distribution networks. Planning of rural lv distribution networks involves the optimal design of radial configuration and location of mv/lv substations considering cost functions. In this work both goals are considered by taking into account different conductors, voltage drop constraints, power losses in lines and deterministic loads. The algorithms developed in this paper are based on dynamic programming; and were implemented on large-scale rural lv distribution networks, but they could also be used for other network optimization.

**Index Terms**—Dynamic programming, low-voltage network planning, network design.

## I. INTRODUCTION

POWER network planning and its current development is a continuous process involving spatial load forecasting, systems analysis and operational evaluations. The importance of the design of lv networks was described by several authors, who suggested theoretical and practical methods to assist in the manual and computer designs of networks. Some of these methods were included in the planning guides of electricity supply industries [1]. The general problem in the planning of lv distribution networks is essentially to search for a radial network with lowest overall cost by taking into account: mv/lv substations (size and location), lv lines (routes and capacities) to supply a given spatial distribution of forecast loads, thermal limits (lines and substations) and voltage level. Different planning practices and processes are needed for urban and rural networks.

Rural networks, as distinct from urban networks, are constrained by space, have slower load densities, have smaller equipment ratings and commonly use overhead lines.

Existing computer methods are based on optimizing or sub-optimizing line costs only. Grimsdale and Sinclair [2] and Chia [3] consider that the total cost is only proportional to the length of network and do not consider power losses and voltage level. Carson and Cornfield [1] proposed an iterative method based on load-flow calculation over a specific graph, radial network selection and tapered radial network design. The method considers continuous cable cost function and produces a series of network configurations. Hindi, Brameller and Hamam [4] present

a nonexhaustive method based on the branch-and-bound transshipment algorithms to the conductor optimization problem [5]. Snelson and Carson [6] proposed solving the conductor optimization problem using continuously tapered distributor criteria and a variable cable cost/voltage drop function. Tram and Wall [7] present a process based on dynamic programming to solve the conductor optimization problem.

In this paper the network planning is carried out in one stage (static planning), so the load models are considered constant. The method presented in this paper is based on the application of dynamic programming optimization on a radial lv distribution network [8]. Thus, the global optimal solution of the large-scale radial lv distribution network is obtained.

The algorithm works over a lv network defined by a connected acyclic graph (tree), where loads are in nodes and cost and constraints of elements are considered. Usually, the specific graph is defined by the minimal spanning tree (minimum Euclidean distance algorithm [8]), another tree could be used, however.

The dynamic programming optimization process (Section III) is based on dividing the tree (lv network) in several sub-trees, and each sub-tree is optimized (Section IV) by considering the model of lv system (Section II). The set of sub-trees produces a forest, and the minimum cost forest,  $F^*$ , is the optimal solution for the lv distribution network problem. So the proposed method for optimal design of lv networks can be divided in three processes: 1) To obtain a forest (sub-trees), 2) To calculate the cost of sub-trees and 3) To search for the minimum cost forest. The forest is obtained by means of an exhaustive algorithm and the sub-trees are represented by oriented-graphs. For each arc of the oriented-graph, the cost is evaluated applying dynamic programming optimization. Finally the minimum cost forest can be obtained.

## II. MODEL OF LOW-VOLTAGE NETWORKS

### A. Topological Structure

The radial structure is the most usually employed in the design of rural low voltage networks. The proposed algorithm uses a spanning tree to search the forest of low voltage radial networks, each one of them with a mv/lv substation (Fig. 1). The best results are obtained by using the minimal Euclidean spanning tree, because the length of the lines is an important term in the cost of rural networks.

The load or consumers are modeled by current sources and dc load flow is used.

Manuscript received July 10, 2000. This work was supported by the Union Fenosa S.A. electric utility.

The authors are with the Departamento de Enxeñaría Eléctrica, Universidade de Vigo, Spain (e-mail: ediaz@le.uvigo; emiguez@le.uvigo; jcidras@le.uvigo).

Publisher Item Identifier S 0885-8950(01)08877-0.

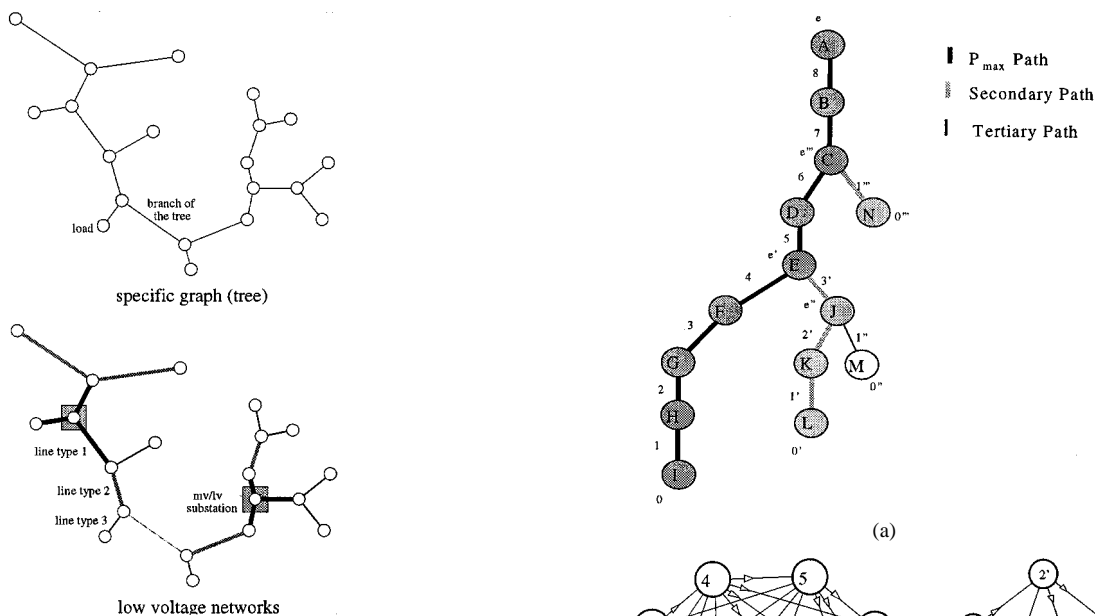


Fig. 1. Topological structure of the network.

### B. Cost and Constraints of Components

**MV/LV Substation Cost:** The cost of the different substations considered in the algorithm are obtained by the following expression:

$$C_{s_k} = c_{s_k} + c'_{s_k} \bullet P_k^2 \quad (1)$$

where

- $c_{s_k}$  investment cost and cost of iron losses in the type  $k$  substation;
- $c'_{s_k}$  cost of electrical losses in the type  $k$  substation;
- $P_k$  real power that flows through the type  $k$  substation;
- $P_k^{\max}$  limit of power of the type  $k$  substation.

If  $P_k > P_k^{\max}$  cost of the type  $k$  substation is considered infinite. The costs for a typical commercial mv/lv substation are shown in Appendix.

**Line Cost:** The line cost (branch cost) of the branch  $(i, j)$  is defined by the expression:

$$C_{(i,j),t_k} = (c_{t_k} + c'_{t_k} \bullet I_{(i,j)}^2) \bullet d_{(i,j)} \quad (2)$$

where

- $t_k$   $k$  type conductor;
- $I_{(i,j)}$  current that flows in branch  $(i, j)$ , from  $i$  node to  $j$  node;
- $d_{(i,j)}$  distance of branch  $(i, j)$ ;
- $c_{t_k}$  cost of the  $k$  type line, per unit of length;
- $c'_{t_k}$  cost of electrical losses in type  $k$  lines, per unit of length;
- $I_t^{\max}$  is the limit of current of the  $k$  type conductor.

If  $I_{(i,j)} > I_t^{\max}$  the cost of the type  $k$  substation is considered infinite. The branch costs for typical Spanish commercial lines are shown in the Appendix.

**Line Voltage-Drop:** The voltage-drop in branch  $(i, j)$  is defined by the expression:

$$\Delta u_{(i,j),t_k} = \Delta u_{t_k} \bullet I_{(i,j)} \bullet d_{(i,j)} \quad (3)$$

where  $\Delta u_{t_k}$  is the voltage-drop of branch  $(i, j)$ , with the  $k$  type line, per unit of length and current.

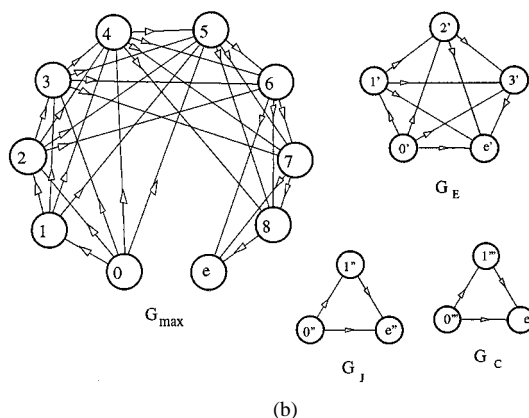


Fig. 2. (a) Initial tree and the longest paths. (b) Oriented graphs associated to longest paths.

### III. PROPOSED ALGORITHM

The initial topology network is a specific tree, where loads are considered in nodes. The typical initial tree is defined by the minimum distance algorithm (minimum spanning tree,  $T$ ) [8]. Using a topological process, the tree  $T$  is divided in several sub-trees; and, consequently, a forest is defined. The topological process and the dynamic algorithm combine to permit a search for the optimal forest from the initial tree  $T$ .

The topological process is based on the following recursive steps:

- 1) Select the longest path (primary path),  $P_{\max}$ , from the tree  $T$ . This primary path,  $P_{\max}$ , is numerically ordered. For Fig. 2(a) the longest path is  $P_{\max} = \{A, B, C, D, E, F, G, H, I\}$ .
- 2) From each bifurcation node of  $P_{\max}$  (i.e.,  $C, E$ ) new longest paths (the secondary paths) are defined (i.e.,  $P_C$  and  $P_E$ ). These secondary paths are defined from a bifurcation node and with nodes not included in  $P_{\max}$  (i.e.,  $P_C = \{C, N\}$   $P_E = \{E, J, K, L\}$ ). This process is repeated with the secondary paths, so resulting in tertiary paths (i.e.,  $P_J = \{J, M\}$ ), fourth level paths, .... The process ends when all bifurcation nodes have been considered.

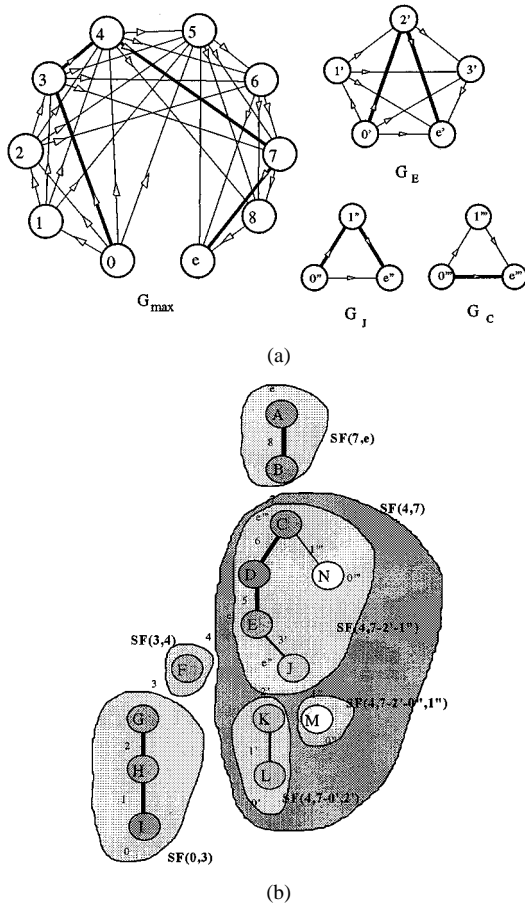


Fig. 3. (a) Routes in oriented-graphs. (b) Sub-trees.

- 3) Build the oriented-graph associated to  $P_{\max}$  path. In this oriented-graph, noted by  $G_{\max}$ , two extreme fictitious vertices (0 and  $e$ ) are included and the vertices are the branches of  $P_{\max}$  path [see Fig. 2(b)]. Other oriented-graphs associated with the secondary, tertiary, ... [i.e.,  $G_E$ ,  $G_C$  and  $G_J$ ] paths are built. [Fig. 2(b).]
- 4) The oriented-graph  $G_{\max}$  is used to search for cuts of  $T$  tree. Thus, a route is defined from initial vertex 0 until terminal vertex  $e$  of  $G_{\max}$ . For example, the route (0-3-4-7- $e$ ) in Fig. 3(a), meaning that branches (3), (4) and (7) of the spanning  $T$  tree are eliminated, and consequently, a forest  $F_{(0,3,4,7,e)}$  of sub-trees is created. In Fig. 3(b) the forest  $F_{(0,3,4,7,e)} = \{\text{sub-trees } SF(0,3), SF(3,4), SF(4,7), SF(7,e)\}$  is shown and Fig. 3(a) shows the route (0-3-4-7- $e$ ) of  $G_{\max}$ , and each arc of  $G_{\max}$  represents a sub-tree of forest  $F_{(0,3,4,7,e)}$ . If any sub-tree of a forest has bifurcation nodes, the secondary, tertiary, ... oriented-graphs  $G_C$ ,  $G_E$  and  $G_J$  are needed, and other routes (sub-sub-trees) can be defined: i.e., ( $0'$ - $2'$ - $e'$ ) ( $0''$ - $e''$ ) ( $0'''$ - $1'''$ - $e'''$ )—Fig. 3(a). So all the sub-trees can be expressed by:  $SF(0,3)$ ,  $SF(3,4)$ ,  $SF(7,e)$ ,  $SF(4,7-0',2')$ ,  $SF(4,7-0'',1'')$  and  $SF(4,7-0''',1''')$ —Fig. 3(b).

#### A. Dynamic Programming

If the sub-tree  $SF_{(i,j)}$  has no bifurcation nodes, the arc  $(i,j)$  cost of  $G_{\max}$ , is the cost of the sub-tree. Otherwise, if

the sub-tree  $SF_{(i,j)}$  has bifurcation nodes, the cost depends on routes defined from secondary, tertiary, ... oriented-graphs. So, the cost of arc  $(i,j)$ ,  $S_{(i,j)}$ , is defined by the expressions:

- i) without bifurcation nodes: (see section Evaluation of a tree)

$$S_{(i,j)} = \min\{\text{cost of } SF_{(i,j)}\}$$

- ii) with bifurcation nodes:

$$S_{(i,j)} = \min \left\{ \begin{array}{l} \text{cost of } SF_{(i,j;\alpha)} / \\ \alpha \text{ are routes of secondary,} \\ \text{tertiary, ... oriented-graphs} \end{array} \right\} \quad (4)$$

When the cost of all arcs of the oriented-graph  $G_{\max}$  are known, the optimal forest  $F^*$  can be expressed as:

$$F^* = \{F_{(x)}/x \text{ is the route of } G_{\max}, \text{ from } 0 \text{ to } e, \text{ of minimum cost}\}.$$

The practical implementation of the proposed method needs the oriented-graphs to be incomplete; otherwise the proposed method can be considered an NP problem. Fortunately, in lv networks no complete oriented-graphs are presented if the following proposition is taken into consideration: "If the arc  $S(i,j)$  has an infinite value, caused by load powers or voltage level drop level, all arcs  $(i,k)$ —from node  $i$  to nodes  $k$  upstream  $j$ —also have infinite values." Mathematically, it can be expressed by:

$$\text{If } S_{(i,j)} = \infty \text{ then } S_{(k,h)} = \infty \quad k/k \leq i, h \geq j.$$

Thus, in Fig. 3(a) if  $S(0,6) = \infty$  then  $S(0,7)$  and  $S(0,8)$  have  $\infty$  value also.

#### IV. EVALUATION OF A TREE

In the process to build the oriented-graph, there is need to evaluate the cost of arcs; where each arc represents a sub-tree and each sub-tree could be formed by other sub-trees. This section describes the algorithm, based on dynamic programming, to evaluate the cost of a generic tree. It is considered that the tree is fed by only one mv/lv substation, and when this is not possible the cost value is infinite. The proposed algorithm considers mv/lv substation location as not fixed, so is an improvement of Tram and Wall's [7] method where mv/lv substation locations are considered fixed.

From a generic tree  $T$ , the subtrees are defined:  $A_{(i,j)}^i$  and  $A_{(i,j)}^j$ ; as a result of taking off the branch  $(i,j)$ . The  $A_{(i,j)}^i$  sub-tree includes the node  $i$ , and  $A_{(i,j)}^j$  the node  $j$ . The current,  $I_{(i,j)}^i$ , that will be named sub-tree current, is defined by the addition of all loads into the sub-tree  $A_{(i,j)}^i$ , and that will feed  $A_{(i,j)}^j$  if the mv/lv substation is in sub-tree  $A_{(i,j)}^j$ . Similarly for current  $I_{(i,j)}^j$  of sub-tree  $A_{(i,j)}^j$ . Fig. 4 presents an example.

By using sub-tree  $A_{(i,j)}^i$ , costs can be represented as a function of voltage drop for the various types of conductors employed along each branch. According to the dynamic

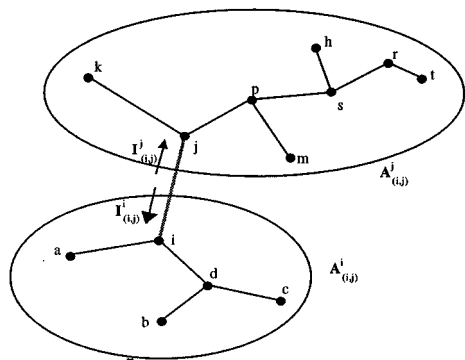


Fig. 4. Tree and sub-trees.

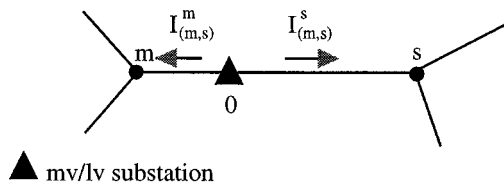


Fig. 5. mv/lv substation location.

programming process the cost function is associated with maximum voltage-drop in the sub-tree.

So mathematically, the cost function of sub-tree  $A^i_{(i,j)}$  can be noted by  $D^i_{(i,j)}[\Delta u^i]$ , where  $\Delta u^i$  is the maximum voltage-drop in sub-tree  $A^i_{(i,j)}$ . Recursively the expression is:

$$D^i_{(i,j)}[\Delta u^i] = \sum_{\substack{x=adj(i) \\ x \neq j}} \min_{t_k \in Y} \left\{ D^x_{(x,i)}[\Delta u^x_s] + C_{(x,i), t_k} \right\} / \Delta u^i = \max \left\{ \Delta u^x_s + \Delta u_{(x,i), t_k} \right\} \quad (5)$$

where

- $adj(i)$  adjacent nodes to  $i$ ;
- $C_{(x,i), t_k}$  cost of branch  $(x, i)$  with the  $t_k$  type conductor defined by expression (2);
- $Y$  of type conductors;
- $\Delta u^x_s$  represents the “ $s$ ” different voltage-drop values in sub-tree  $A^x_{(x,i)}$ .

The process starts in leaf nodes,  $f$ , where  $A^f_{(f,.)}[0] = 0$  and  $\Delta u^f_0 = 0$  are considered. When the recursive process is ended, each node has an associated table: DV-table. The DV-table is defined by cost and voltage-drop. So each node  $i$  has  $\delta$  tables, where  $\delta$  is the degree of node  $i$  (number of branches which have  $i$  as terminal endpoint). These DV-tables have a dimension according to level in the tree and number of types of lines used.

The operation process to calculate the cost of  $T$  tree can be defined by the following algorithm:

- 1) Select one branch  $(p, s)$  where it is supposed the mv/lv substation exists.
- 2) Select D-tables corresponding to  $A^m_{(r,s)}$  and  $A^s_{(r,s)}$  sub-trees:  $D^m_{(r,s)}$  and  $D^s_{(r,s)}$ .
- 3) The cost  $S_{(m,0,s)}$  of  $T$ , considering the substation in fictitious node “0” (see Fig. 5), is determined as:

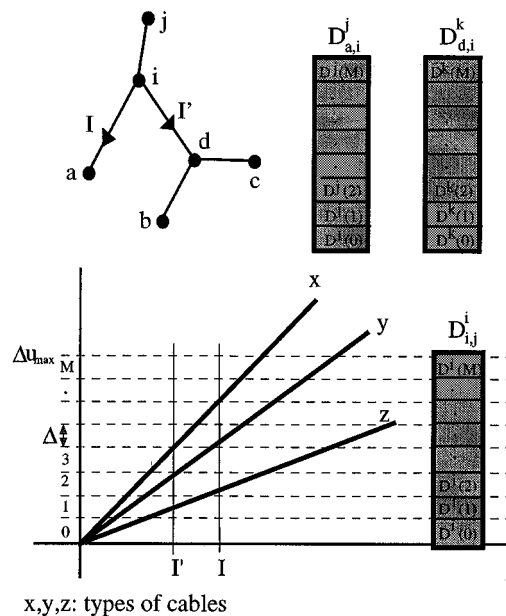


Fig. 6. DV-table building.

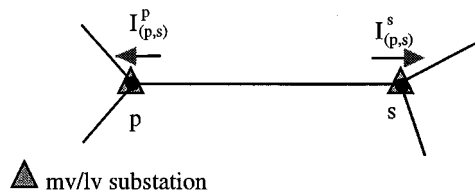


Fig. 7. Optimal location of mv/lv substation.

$$S_{(m,0,s)} = C_{subst} + \min \left\{ \begin{aligned} & D^m_{(m,s)}[p] + D^s_{(m,s)}[q] \\ & + C_{(m,0), t_k} + C_{(0,s), t_{k'}} / \\ & \max \left\{ p \cdot \Delta u + \Delta u_{(m,0), t_k}, q \cdot \Delta u \right. \\ & \left. + \Delta u_{(0,s), t_{k'}} \right\} \leq \Delta u_{max} \\ & p, q = 0, \dots, M \end{aligned} \right\} \quad (6)$$

where

- “0” represents the fictitious node (between  $p$  and  $s$ ) where the mv/lv substation is located;
- $\Delta u_{max}$  maximum voltage-drop submitted.

To search for the node “0,” where the mv/lv substation is located, a discretization of branch  $(m, s)$  is needed. So the length of branch  $(m, s)$  is discretized in  $N$  intervals:  $(m, 1), (1, 2), \dots (N - 1, s)$ ; and each new node  $(1, 2, \dots N - 1)$  is a new location of the mv/lv substation (node “0”) to evaluate.

### V. IMPROVEMENT OF OPTIMIZATION PROCESS

To improve the optimization process two actions can be considered. The first improvement is associated with DV-table reduction [7], and the second, with location of the mv/lv substation.

DV-tables can be reduced by the discretization of voltage-drop instead of considering the type of lines

(Appendix). So, for  $M$  intervals of  $\Delta u_{\max}$  and  $I$  current in the branch, the DV-table has  $M$  rows. Fig. 6 shows an example of building DV-tables.

The elements of DV-tables are defined by the following recursive process, where  $p$  is the  $p$  row of DV-table: ( $p \cdot \Delta = \Delta u_p$ )

$$D_{(i,j)}^i(p) = \min \left\{ \begin{array}{l} D_{(a,i)}^a(\alpha) + c_x, D_{(a,i)}^a(\beta) + c_y, D_{(a,i)}^a(\gamma) + c_z / \\ (\alpha \bullet \Delta + \Delta u_x \leq p \bullet \Delta, \\ \beta \bullet \Delta + \Delta u_y \leq p \bullet \Delta, \\ \gamma \bullet \Delta + \Delta u_z \leq p \bullet \Delta) \\ \alpha, \beta, \gamma = p, p-1, \dots, 0 \end{array} \right\} + \min \left\{ \begin{array}{l} D_{(d,i)}^d(\alpha) + c'_x, D_{(d,i)}^d(\beta) + c'_y, D_{(d,i)}^d(\gamma) + c'_z / \\ (\alpha \bullet \Delta + \Delta u'_x \leq p \bullet \Delta, \\ \beta \bullet \Delta + \Delta u'_y \leq p \bullet \Delta, \\ \gamma \bullet \Delta + \Delta u'_z \leq p \bullet \Delta) \\ \alpha, \beta, \gamma = p, p-1, \dots, 0 \end{array} \right\} \quad (7)$$

where

$c_x, c_y$ and $c_z$	costs of branch $(i, j)$ with $x, y$ and $z$ lines, respectively;
$c'_x, c'_y$ and $c'_z$	costs of branch $(i, k)$ with $x, y$ and $z$ lines, respectively;
$\Delta u_x, \Delta u_y$ and $\Delta u_z$	voltage-drops in branch $(i, j)$ with $x, y$ and $z$ lines, respectively;
$\Delta u'_x, \Delta u'_y$ and $\Delta u'_z$	voltage-drops in branch $(i, k)$ with $x, y$ and $z$ lines, respectively;
$\Delta$	$= \Delta u_{\max}/M$ ;
$\alpha, \beta, \gamma$	generic intervals from 0 to $M$ .

In (7), if a constraint is not met, i.e.,  $\alpha \cdot \Delta + \Delta u_x > p \cdot \Delta$ , its associated value is infinite ( $D(\alpha) + c_x = \infty$ ).

The proposed optimization process is based on the following propositions:

**Proposition 1:** If in the optimization of tree  $T_\alpha$  voltage-drops are not considered, D-tables have only one element (Appendix).

**Proposition 2:** If in optimization of tree  $T$  voltage-drops are not considered, the mv/lv substation has its optimal location in a node (see Fig. 7); and this node (called highest current node), for a specific branch  $(p, s)$ , has the highest sub-tree current:  $\max\{I_{(p,s)}^s, I_{(p,s)}^p\}$ .

Also, it is possible to prove that the branch  $(p, s)$ , where the location of the mv/lv substation is optimal when voltage-drops are not considered, can be defined mathematically by the expression:

$$\min \left\{ \left| I_{(p,s)}^p - I_{(p,s)}^s \right| / p, s \in T \right\}. \quad (8)$$

In conclusion, if in the tree  $T$  voltage-drops are not considered, the process of optimization is reduced to searching for the minimum  $\{D_{(p,s)}^s + C(p, s) + D_{(p,s)}^p, D_{(p,s)}^p + C(s, p) + D_{(p,s)}^s\}$ . Also, the cost  $S$  of  $T$  is the minimum possible for this sub-tree. So some solutions, including voltage-drops, will always have a higher cost;  $S_{\min}$  is a "minimum" solution.

**Proposition 3:** If in the optimization of tree  $T$  voltage-drops are not considered, and the mv/lv substation is located in a node

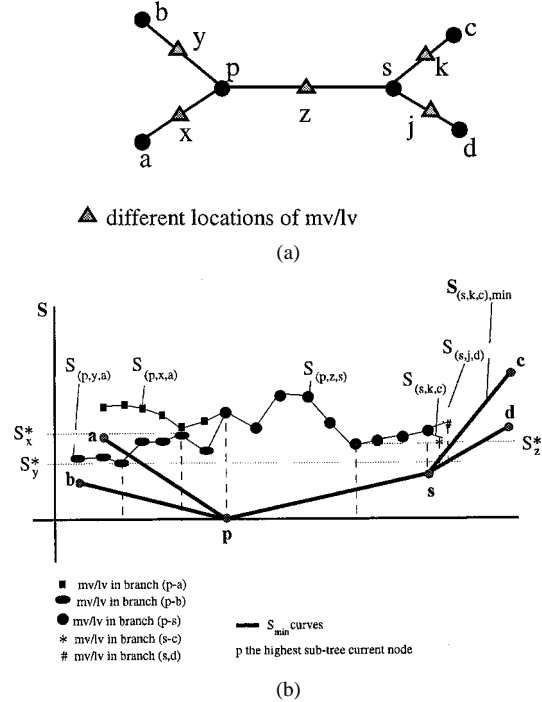


Fig. 8. (a) Example tree. (b) Optimization process applied to Fig. 8(a).

or branch (highest current node, node "0" in Fig. 5), for any generic node  $x$ ,  $S_x > S_{\min}$  and  $S_x$  has a recursive linear dependence on the distance from the highest current node.

**Proposition 4:** On the other hand, if in the optimization of tree  $T$  only the lines of maximum size are considered, the optimal solution  $S_{\max}$ , if it exists, corresponds to "maximum" solution. So, any solution,  $S_\alpha$ , including voltage-drops, will be "bounded" by:  $S_{\min} < S_\alpha < S_{\max}$ .

Using proposition 4, the optimization process to calculate the cost of tree  $T$  can be improved by a recursive algorithm. This algorithm will be described using the tree in Fig. 8(a), and its results are shown in Fig. 8(b):

- i) Calculate  $S_{\min}$  curves, defined by proposition 2)
- ii) Search the highest sub-tree current node ( $S_{\min}^*$ : optimal location of mv/lv substation for  $S_{\min}$  problem): Node  $p$
- iii) Search the adjacent nodes to  $p$  node:  $s, a, b$
- iv) Calculate the tree costs:

$$S_{(p,x,a)}, S_{(p,y,b)}, S_{(p,z,s)}$$

where:  $x, y$  and  $z$  are different locations of mv/lv substation.

- v) Search:

$$\min \{S_{(p,x,a)}, S_{(p,y,b)}, S_{(p,z,s)}, S_y^*\} = S^*$$

- vi) Search the adjacent nodes to  $s, a$  and  $b$  nodes:  $c, d$
- vii) If  $S_{(s,k,c), \min} < S^*$  then do:  $S_{(s,k,c)}$   
Else Nothing  
If  $S_{(s,j,d), \min} < S^*$  then do:  $S_{(s,j,d)}$   
Else Nothing

where:  $k$  and  $j$  are different locations of mv/lv substation.

TABLE I  
RESULTS OF LV NETWORKS FOR 5 DIFFERENT AREAS

area (km <sup>2</sup> )	n° of loads	total load (kW)	lv lines (km)	n° of subst	tot. cost (Mpts)
7876	9162	411584	786	6207	13531
9799	7606	146614	592	5515	11702
10565	5438	422383	290	4318	8642
7278	3344	177181	179	2651	5307
4477	5597	312507	425	3718	7968

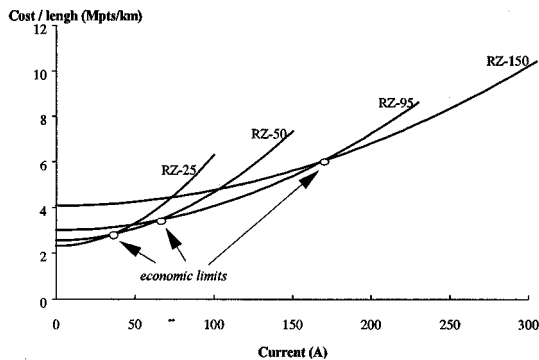


Fig. 9. The costs for typical Spanish commercial lines.

TABLE II  
THE COSTS FOR COMMERCIAL mv/LV SUBSTATIONS

mv/lv type Rating transformer (kVA)	$c_s$ (Mpts)	$c_s$ (pts/kVA <sup>2</sup> )
25	1.484	163.74
50	1.572	64.32
100	1.820	25.58
160	2.277	13.42
250	3.870	7.602
400	4.434	4.203
630	5.070	2.394
1000	5.763	1.535

viii) Search:

$$\min \{S^*, S_{(s,k,c)}, S_{(s,j,d)}, S_{(p,z,s)}\} = S^*$$

ix) End when all branches are explored

## VI. RESULTS

The proposed method was applied to several large lv networks. Table I shows the results for 5 areas with different characteristics.

## VII. CONCLUSION

A method to plan a low-voltage radial distribution network was developed. The proposed method is based on implementation of dynamic programming optimization on a specific tree, considering: several conductors, voltage drops and exact cost

function of lines and mv/lv substations. The proposed method does not need to make any simplification of the functions of cost and restrictions, as in the methods of references [2]–[6].

With the goal of applying the proposed method to large-scale rural lv network planning, several improvements were developed. Although the specific can be any, in this paper the minimum Euclidean distance tree was used.

## APPENDIX

The line cost curves (Fig. 9) and the substation cost values (Table II) were calculated considering: 7 pts/kW-losses, 25-year planning, 25% overload factor, 1% annual inflation and 5% annual interest. The annual loss load factor employed was:  $l_s f = 0.16 * l f + 0.84 * l f^2$  (proposed by salis [9] by rural networks), with the load factor  $l f = 0.25$ .

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**Eloy Díaz-Dorado** received the Ph.D. degree in electrical engineering from the Universidade de Vigo (Spain) in 1999. He is a lecturer of the Departamento de Enxeñaría Eléctrica of the Universidade de Vigo and he is interested in planning and estimation of power systems.

**Edelmiro Miguez** received the Ph.D. degree in electrical engineering from the Universidade de Vigo (Spain) in 1999. He is a lecturer of the Departamento de Enxeñaría Eléctrica of the Universidade de Vigo and he is interested in planning and analysis of distribution systems.

**José Cidrás** received the Ph.D. degree in electrical engineering from the Universidade de Santiago de Compostela (Spain) in 1987. He is professor and head of the Departamento de Enxeñaría Eléctrica of the Universidade de Vigo (Spain) and leads some investigation projects into wind energy, photovoltaic energy and planning of power systems.