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A third order model for the doubly-fed induction machine

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Abstract

Induction generators are generally simulated by means of a well-known model described by Brereton et al. [1], based on the induction motor equations derived by Stanley [2]. In this model the possibility of opening the rotor circuit in order to inject a voltage source is not taken into account, although there are other models where it is dealt with [3]. This paper presents an alternative way of obtaining the mentioned model and introduces the possibility of modeling voltage sources in the rotor circuit, which can be very useful when simulating some generating schemes, such as variable speed asynchronous wind turbines.

Keywords: Induction machine models; Stator; Rotor

1. Introduction

Induction generators are the most commonly used electric generators in industry, and they are particularly common in some renewable energy applications such as wind turbines (WTs). Several models have been proposed to simulate them. The model proposed by Brereton et al. is of great use when induction generators must be dynamically simulated. This model is a very valid one when the machine works with its rotor short-circuited, that is, the rotor voltage having a value of 0. The trend for future years seems to be the use of variable-speed WTs, in order to achieve an optimization of the generated power profiles. Several solutions have been proposed, such as the use of synchronous generators linked to the electrical network through electronic devices, and doubly-fed induction generators, which this paper is concerned with. The simulation of doubly-fed induction generators has been dealt with in many papers, and generally involves handling a set of differential equations related to the rotor and stator currents, fluxes and voltages. The purpose of this paper is to obtain a set of more simplified equations than those generally used.

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2. The third order models of the induction machine

First, the matrices k_s and k_r [4,5] will be assumed, which allows one to formulate a change of variables transforming them from the three-phase variables system to an arbitrary reference frame. In this case the new reference frame is rotating at synchronous speed. Both matrices can be seen in Appendix A.

The following notation will be employed:

- I_{r123} is a vector whose components are the per phase rotor currents ($I^T_{r123} = (I_{r1} I_{r2} I_{r3})$), and I_{rqd0} ($I^T_{rqd0} = (I_{rq} I_{rd} I_{r0})$) is a vector whose components are the rotor currents seen from the $dq0$ axes. The superindex T is used for notating transpose matrices.
- I_{sabc} and I_{sqd0} are the stator currents in both reference frames.
- V_{r123} and V_{rqd0} are the rotor voltages.
- V_{sabc} and V_{sqd0} are the stator voltages.
- φ_{r123} and φ_{rqd0} are the rotor fluxes.
- φ_{sabc} and φ_{sqd0} are the stator fluxes.

The following equations express the relationship between both reference frames:

$$V_{rqd0} = k_r V_{r123} \Rightarrow V_{r123} = k_r^{-1} V_{rqd0} \tag{1a}$$

$$V_{sqd0} = k_s V_{sabc} \Rightarrow V_{sabc} = k_s^{-1} V_{sqd0} \tag{1b}$$

$$I_{rqd0} = k_r I_{r123} \Rightarrow I_{r123} = k_r^{-1} I_{rqd0} \tag{1c}$$

$$I_{sqd0} = k_s I_{sabc} \Rightarrow I_{sabc} = k_s^{-1} I_{sqd0} \tag{1d}$$

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$$\varphi_{rqd0} = k_r \varphi_{r123} \Rightarrow \varphi_{r123} = k_r^{-1} \varphi_{rqd0} \quad (1e)$$

$$\varphi_{sqd0} = k_s \varphi_{sabc} \Rightarrow \varphi_{sabc} = k_s^{-1} \varphi_{sqd0} \quad (1f)$$

where the matrices k_r and k_s are given in Appendix A, where both reference frames can also be seen.

In order to obtain the model, the following equations, valid for the rotor and stator electrical circuits, must be taken into account:

$$\begin{pmatrix} V_{r123} \\ V_{sabc} \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_{r123} \\ I_{sabc} \end{pmatrix} + \frac{d}{dt} \begin{pmatrix} \varphi_{r123} \\ \varphi_{sabc} \end{pmatrix} \quad (2)$$

where

$$r = \begin{bmatrix} R_R & 0 & 0 \\ 0 & R_R & 0 \\ 0 & 0 & R_R \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix},$$

R_R is the rotor resistance and R_s the stator resistance. The machine is assumed to be balanced. Taking this assumption into account, the component '0' can be neglected. So, from now on only the 'd' and 'q' components are employed.

Changing the reference frame by means of the matrices

$$\begin{pmatrix} k_r & 0 \\ 0 & k_s \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} k_r & 0 \\ 0 & k_s \end{pmatrix}^{-1},$$

the following is obtained:

$$\begin{pmatrix} V_{rqd} \\ V_{sqd} \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_{rqd} \\ I_{sqd} \end{pmatrix} + \begin{pmatrix} k_r & 0 \\ 0 & k_s \end{pmatrix} \frac{d}{dt} \left(\begin{pmatrix} k_r^{-1} & 0 \\ 0 & k_s^{-1} \end{pmatrix} \begin{pmatrix} \varphi_{rqd} \\ \varphi_{sqd} \end{pmatrix} \right) \quad (3)$$

2.1. Model for the induction machine with short-circuited rotor

In the case of an induction generator with short-circuited rotor, which is a common solution in constant speed WTs , the previous equations become the following, by neglecting the stator transients $d\varphi_{sqd}/dt = 0$:

$$\begin{pmatrix} 0 \\ V_{sqd} \end{pmatrix} = \begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_{rqd} \\ I_{sqd} \end{pmatrix} + \begin{bmatrix} k_r \dfrac{dk_r^{-1}}{dt} & 0 \\ 0 & k_s \dfrac{dk_s^{-1}}{dt} \end{bmatrix} \begin{pmatrix} \varphi_{rqd} \\ \varphi_{sqd} \end{pmatrix} + \begin{bmatrix} \dfrac{d\varphi_{rqd}}{dt} \\ 0 \end{bmatrix} \quad (4)$$

The rotor and stator fluxes can be written as functions of their currents, as follows:

$$\begin{pmatrix} \varphi_{rqd} \\ \varphi_{sqd} \end{pmatrix} = \begin{pmatrix} L_{rr} & L_{rs} \\ L_{rs} & L_{ss} \end{pmatrix} \begin{pmatrix} I_{rqd} \\ I_{sqd} \end{pmatrix} \quad (5)$$

where:

$$L_{rr} = \begin{pmatrix} l_{rr} & 0 \\ 0 & l_{rr} \end{pmatrix} \quad (6a)$$

$$L_{ss} = \begin{pmatrix} l_{ss} & 0 \\ 0 & l_{ss} \end{pmatrix} \quad (6b)$$

$$L_{rs} = \begin{pmatrix} l_{rs} & 0 \\ 0 & l_{rs} \end{pmatrix} \quad (6c)$$

and:

- $l_{rr} = l_2 + l_{rs}$, and l_2 is the leakage inductance in a phase of the rotor.
- $l_{ss} = l_1 + l_{rs}$, and l_1 is the stator leakage inductance.
- l_{rs} is the magnetizing inductance.

From Eq. (5) the following expression for the rotor currents can be obtained:

$$I_{rqd} = L_{rr}^{-1}(\varphi_{rqd} - L_{rs} I_{sqd}) \quad (7)$$

Now the rotor short-circuit constant is defined as $T_0' = l_{rr}/\mathrm{r}$. So from Eq. (4) the following equation can be obtained:

$$\frac{d\varphi_{rqd}}{dt} = -k_r \frac{dk_r^{-1}}{dt} \varphi_{rqd} - \frac{1}{T_0'}(\varphi_{rqd} - L_{rs} I_{sqd}) \quad (8)$$

On the other hand, from Eqs. (5) and (7) the stator flux can also be written as:

$$\begin{aligned} \varphi_{sqd} &= L_{rs} I_{rqd} + L_{ss} I_{sqd} \\ &= L_{rs} L_{rr}^{-1} \varphi_{rqd} + (L_{ss} - L_{rs} L_{rr}^{-1} L_{rs}) I_{sqd} \end{aligned} \quad (9)$$

and also from Eqs. (4) and (9) the next equation for the stator voltage is valid:

$$V_{sqd} = R I_{sqd} + k_s \frac{dk_s^{-1}}{dt} (L_{rs} L_{rr}^{-1} \varphi_{rqd} + (L_{ss} - L_{rs} L_{rr}^{-1} L_{rs}) I_{sqd}) \quad (10)$$

For the sake of simplicity the following variables and constants are defined, $\varphi'_{rqd} = L_{rs}L_{rr}^{-1}\varphi_{rqd}$, $L' = L_{ss} - L_{rs}L_{rr}^{-1}L_{rs}$ and $L = L_{rs}$ With regard to this and to Eq. (8) the following equations can be written:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} = -\begin{pmatrix}0 & -s\omega_s\\ s\omega_s & 0\end{pmatrix}\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} - \frac{1}{T'_0}\left(\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} - \begin{pmatrix}l-l' & 0\\ 0 & l-l'\end{pmatrix}\begin{pmatrix}I_{sq}\\ I_{sd}\end{pmatrix}\right) \tag{11}$$

And with the same variables and taking Eq. (10) into account:

$$\begin{pmatrix}V_{sq}\\ V_{sd}\end{pmatrix} - \begin{pmatrix}0 & -\omega_s\\ \omega_s & 0\end{pmatrix}\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} = \begin{pmatrix}R & 0\\ 0 & R\end{pmatrix}\begin{pmatrix}I_{sq}\\ I_{sd}\end{pmatrix} \tag{12}$$

where the values of l and l' are derived in Appendix B and the expression of slip has been taken into account ($s = \omega_s - \Omega/\omega_s$).

Now, applying the next change of variables:

$$E'_{qd} = \begin{pmatrix}0 & -\omega_s\\ \omega_s & 0\end{pmatrix}\varphi'_{rqd} \tag{13}$$

the equations can be finally expressed as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}E'_q\\ E'_d\end{pmatrix} = -\begin{pmatrix}0 & -s\omega_s\\ s\omega_s & 0\end{pmatrix}\begin{pmatrix}E'_q\\ E'_d\end{pmatrix} - \frac{1}{T'_0}\left(\begin{pmatrix}E'_q\\ E'_d\end{pmatrix} - \begin{pmatrix}0 & -(X-X')\\ X-X' & 0\end{pmatrix}\begin{pmatrix}I_{sq}\\ I_{sd}\end{pmatrix}\right) \tag{14}$$

$$\begin{pmatrix}V_{sq}\\ V_{sd}\end{pmatrix} - \begin{pmatrix}E'_q\\ E'_d\end{pmatrix} = \begin{pmatrix}R & 0\\ 0 & R\end{pmatrix}\begin{pmatrix}I_{sq}\\ I_{sd}\end{pmatrix} + \begin{pmatrix}0 & -X'\\ X' & 0\end{pmatrix}\begin{pmatrix}I_{sq}\\ I_{sd}\end{pmatrix} \tag{15}$$

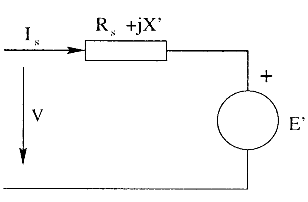

Fig. 1. Induction machine dynamic model.

Now, defining the following complex numbers: $\bar{V}_s = V_{sq} + jV_{sd}$, $\bar{E}' = E'_q + jE'_d$ and $\bar{I}_s = I_{sq} + jI_{sd}$, the equations describing the model are:

$$\frac{\mathrm{d}\bar{E}'}{\mathrm{d}t} = -js\omega_s\bar{E}' - \frac{1}{T'_0}(\bar{E}' - j(X-X')\bar{I}_s) \tag{16a}$$

$$\bar{V}_s - \bar{E}' = (R_s + jX')\bar{I}_s \tag{16b}$$

These equations are the same as those obtained by Brereton et al. [1], and are generally accepted and widely used for transient stability studies, where induction machines are present. The representation of the machine as an electrical circuit can be seen in Fig. 1.

2.2. The proposed model for the doubly-fed induction machine

The case of the doubly-fed induction machine is similar to the previous one, with the difference that in Eq. (4), the rotor voltage is different from 0. The same assumption is made, which means that the stator transients can be neglected. The following is valid when assuming waves of the fundamental frequency only. If this assumption were not made, the derivation of the model would be more complex, due to the difficulty of dealing with harmonic currents when dynamic processes are being analyzed. With regard to this, it can be said that the rotor current harmonics, when reflected to the stator, induce distortion harmonic currents in the stator windings. These currents have relatively low magnitudes, and are not so serious if the rotor is connected to a high-frequency PWM converter. This issue is discussed in [6].

$$\begin{pmatrix}V_{rqd}\\ V_{sqd}\end{pmatrix} = \begin{pmatrix}r & 0\\ 0 & R\end{pmatrix}\begin{pmatrix}I_{rqd}\\ I_{sqd}\end{pmatrix} + \begin{bmatrix}k_r\dfrac{\mathrm{d}k_r^{-1}}{\mathrm{d}t} & 0\\ 0 & k_s\dfrac{\mathrm{d}k_s^{-1}}{\mathrm{d}t}\end{bmatrix}\begin{pmatrix}\varphi_{rqd}\\ \varphi_{sqd}\end{pmatrix} + \begin{bmatrix}\dfrac{\mathrm{d}\varphi_{rqd}}{\mathrm{d}t}\\ 0\end{bmatrix} \tag{17}$$

In this case, by performing the following change of variables:

$$V'_{rqd} = L_{rs}L_{rr}^{-1}V_{rqd} \tag{18}$$

from Eq. (4) the rotor flux equations become:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} = \begin{pmatrix}V'_{rq}\\ V'_{rd}\end{pmatrix} - \begin{pmatrix}0 & -s\omega_s\\ s\omega_s & 0\end{pmatrix}\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} - \frac{1}{T'_0}\left(\begin{pmatrix}\varphi'_{rq}\\ \varphi'_{rd}\end{pmatrix} - \begin{pmatrix}l-l' & 0\\ 0 & l-l'\end{pmatrix}\begin{pmatrix}I_{sq}\\ I_{sd}\end{pmatrix}\right) \tag{19}$$

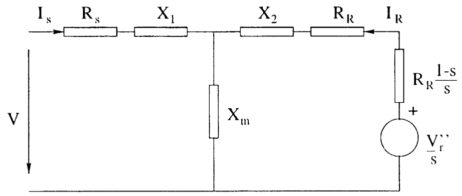

Fig. 2. Doubly-fed induction machine steady-state model.

so, including Eq. (13) the following system is obtained:

$$\frac{d}{dt}\begin{pmatrix} E'_q \\ E'_d \end{pmatrix} = \begin{pmatrix} 0 & -\omega_s \\ \omega_s & 0 \end{pmatrix}\begin{pmatrix} V'_{rq} \\ V'_{rd} \end{pmatrix} - \begin{pmatrix} 0 & -s\omega_s \\ s\omega_s & 0 \end{pmatrix}\begin{pmatrix} E'_q \\ E'_d \end{pmatrix} - \frac{1}{T'_0}\left(\begin{pmatrix} E'_q \\ E'_d \end{pmatrix} - \begin{pmatrix} 0 & -(X-X') \\ X-X' & 0 \end{pmatrix}\begin{pmatrix} I_{sq} \\ I_{sd} \end{pmatrix}\right) \tag{20}$$

and from Eq. (3):

$$\begin{pmatrix} V_{sq} \\ V_{sd} \end{pmatrix} - \begin{pmatrix} E'_q \\ E'_d \end{pmatrix} = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix}\begin{pmatrix} I_{sq} \\ I_{sd} \end{pmatrix} + \begin{pmatrix} 0 & -X' \\ X' & 0 \end{pmatrix}\begin{pmatrix} I_{sq} \\ I_{sd} \end{pmatrix} \tag{21}$$

Finally, the previous equations can be written in a simpler way as follows, by defining the complex number $\bar{V}'_r = V'_{rq} + jV'_{rd}$:

$$\frac{d\bar{E}'}{dt} = j\omega_s\bar{V}'_r - js\omega_s\bar{E}' - \frac{1}{T'_0}(\bar{E}' - j(X-X')\bar{I}_s) \tag{22a}$$

$$\bar{V}_s - \bar{E}' = (R_s + jX')\bar{I}_s \tag{22b}$$

This model is valid for the doubly-fed induction machine, and the electrical circuit of Fig. 1 can represent the machine. So, the case of the induction machine having short-circuited rotor is a special case of the general case, considering the doubly-fed induction machine as the general case.

2.3. Steady-state

In order to obtain the steady-state models, it may be assumed that $d\bar{E}'/dt = 0$. By doing this in the general model, the steady-state circuit can be given for the induction machine. This circuit is represented in Fig. 2, and the reference of the rotor current is chosen to be in concordance to that chosen for the dynamic model.

The equivalence between steady-state and dynamic model is achieved taking the following into account (see Appendix C):

$$\bar{V}''_r = \frac{X_2 + X_m}{X_m}\bar{V}'_r \tag{23}$$

which coincides with $\bar{V}_r = V_{rq} + jV_{rd}$.

Also by doing $d\bar{E}'/dt = 0$, the expression of the voltage source $\bar{E}'$ in steady-state can be obtained and expressed as follows:

$$\bar{E}' = \frac{X_m}{s(X_2 + X_m)}(\bar{V}'_r - R_R\bar{I}_r) \tag{24}$$

which, obviously, in the case that the machine is not doubly-fed, can be written as:

$$\bar{E}' = -\frac{X_m}{s(X_2 + X_m)}R_R\bar{I}_r \tag{25}$$

3. Electromechanical equations and rotor currents

The mechanical power, P_m, electrical power, P_E, and slip, s, are related by means of the electromechanical [7]:

$$\frac{P_m}{1-s} - P_E = -2H\frac{ds}{dt} \tag{26}$$

where H is the inertia constant, proportional to the moment of inertia (J), and defined as the relationship between the energy of the machine at synchronous speed and its rated power.

The term P_E can be calculated as a function of $\bar{E}'$ and $\bar{I}_s$ as follows:

$$P_E = -Re\{\bar{E}'\bar{I}^*_s\} \tag{27}$$

where Re denotes the real part of a complex number.

When the steady-state is reached, the value of P_E so calculated, in the case of the rotor cascade angle having a value of π, coincides with the following expression:

$$P_E = -I_r^2\frac{R_R}{s} - \frac{V'_r I_r}{s} \tag{28}$$

and in the case of the machine with short-circuited rotor, with:

$$P_E = -I_r^2\frac{R_R}{s} \tag{29}$$

If the angle is different from π or 0 the following expression should be written:

$$P_E = -I_r^2\frac{R_R}{s} + \frac{V''_r I_r \cos\phi}{s} \tag{30}$$

where ϕ is the angle between rotor voltage and current.

The following observation must be made, that concern the way of calculating the rotor current. There are some differences between steady-state and dynamic models. For the steady-state model, according to the references given in Fig. 2, the equation is as follows:

$$\bar{I}_r = \frac{(R_s + j(X_1 + X_m))\bar{V}''_r - jsX_m\bar{V}}{(R_s + j(X_1 + X_m))(R_R + js(X_2 + X_m)) + sX_m^2} \tag{31}$$

As for the dynamic model, from Eq. (7), taking into account the fact that $\varphi'_{rqd} = L_{rs}L_{rr}^{-1}\varphi_{rqd}$ and also Eq. (13) the rotor current can be calculated as:

$$\bar{I}_r = \frac{\bar{E}'}{jX_m} - \frac{X_m}{X_2 + X_m}\bar{I}_s \tag{32}$$

which only coincides with that given by Eq. (31) when the machine reaches the steady-state. In spite of this observation, the rotor current in the dynamic model must never be calculated during the simulation.

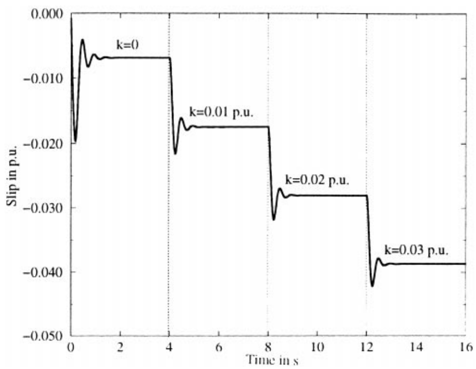

Fig. 3. Evolution of slip during starting and changing rotor voltage.

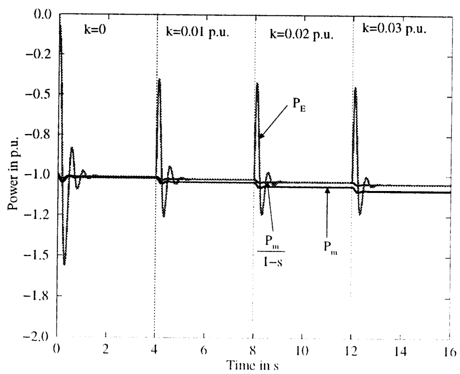

Fig. 4. Evolution of real and mechanical power during starting and changing rotor voltage.

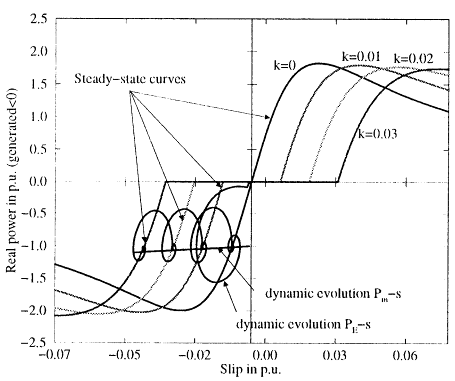

Fig. 5. Real power against slip curves for the doubly-fed induction machine in dynamic and steady-state.

4. Simulations

Simulations have been carried out with the model of an induction generator corresponding to a WT having the parameters given in Appendix D. It is a horizontal axis three-blade asynchronous WT.

For the simulation, the mechanical power is calculated from the wind speed according to the equation

$P_m = 1/2\rho A U^3 c_p$ [8], where ρ is the density of air (1.225 kg m^{-3}), A is the area swept by the rotor, U is the wind speed in m s^{-1} and c_p is the power coefficient. The power coefficient depends on the tip speed ratio, λ, which is the relationship between the speed of the blades and the wind speed. Generally, the power coefficient can be expressed as a polynomial, $c_p = \Sigma\, a_i\lambda^i$. The machine simulated here has the features described in Appendix D.

The control system was not simulated, as it is not the purpose of the paper to study how it works. So, the simulation consists of the starting of the machine and the injection in the rotor circuit, 4 s later, of a voltage source with a value of 0.01 p.u., and an angle of π radians with respect to the rotor current shown in Fig. 2. As a consequence, the evolution of the machine is towards a new starting point.

In the simulation, the constant voltage injected in the rotor is defined as proportional to the stator voltage, V'_r (kV). The electronic devices allow control not only of the rms value but also of the angle of the rotor voltage. From the point of view of the machine, this can be simulated as a voltage source with a magnitude and an angle.

In Fig. 3 the evolution of the slip can be seen. In Fig. 4 the evolution of mechanical and electrical power can be seen. The change in the value of the mechanical power is due to the fact that, with the change of the slip, the tip speed ratio reaches another value and, consequently, the power coefficient and the electrical power do the same. The steady-state, real power-slip curves are shown in Fig. 5. In this curve the evolution of electrical power and slip can also be seen. It can be observed here, that when a new steady-state is reached, the results obtained with the dynamic model are coincident with those obtained with the steady-state one.

5. Conclusions

A model is presented in order to make it easier to dynamically simulate doubly-fed induction machines. Simulations are presented to prove that the model is adequate from the point of view of steady-state. The advantage of the model is that it allows one to deal with the machine with only one differential equation in the electrical part.

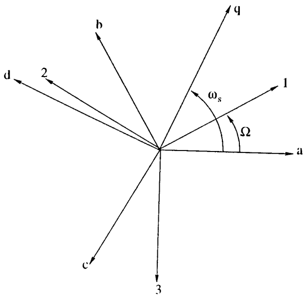

Fig. 6. Stator, rotor and dq reference frames.

One of the assumptions made to derive the model is the fact that the harmonics of electrical magnitudes are neglected. This assumption is generally accepted in many applications involving electrical machines and electronic devices. Nevertheless, the authors think that it would be interesting to study a model able to deal with harmonics.

Appendix A. Matrices used for the transformation of the reference frames

In the matrices the following notation and the reference frames shown in Fig. 6 are used:

$$\Delta\omega = \omega_s - \Omega \tag{33}$$

where:

- ω_s is the synchronous speed.
- $\Omega = p\Omega_R$, with p being the number of pole pairs and Ω_R the rotor speed.

$$k_s = \frac{2}{3}\begin{bmatrix} \cos\omega_s t & \cos\left(\omega_s t - \frac{2\pi}{3}\right) & \cos\left(\omega_s t + \frac{2\pi}{3}\right) \\ -\sin\omega_s t & -\sin\left(\omega_s t - \frac{2\pi}{3}\right) & -\sin\left(\omega_s t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{34}$$

$$k_r = \frac{2}{3}\begin{bmatrix} \cos\Delta\omega t & \cos\left(\Delta\omega t - \frac{2\pi}{3}\right) & \cos\left(\Delta\omega t + \frac{2\pi}{3}\right) \\ -\sin\Delta\omega t & -\sin\left(\Delta\omega t - \frac{2\pi}{3}\right) & -\sin\left(\Delta\omega t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{35}$$

$$k_s^{-1} = \begin{bmatrix} \cos\omega_s t & -\sin\omega_s t & 1 \\ \cos\left(\omega_s t - \frac{2\pi}{3}\right) & -\sin\left(\omega_s t - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\omega_s t + \frac{2\pi}{3}\right) & -\sin\left(\omega_s t + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \tag{36}$$

$$k_r^{-1} = \begin{bmatrix} \cos\Delta\omega t & -\sin\Delta\omega t & 1 \\ \cos\left(\Delta\omega t - \frac{2\pi}{3}\right) & -\sin\left(\Delta\omega t - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\Delta\omega t + \frac{2\pi}{3}\right) & -\sin\left(\Delta\omega t + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \tag{37}$$

$$\frac{\mathrm{d}k_s^{-1}}{\mathrm{d}t} = \omega_s\begin{bmatrix} -\sin\omega_s t & -\cos\omega_s t & 0 \\ -\sin\left(\omega_s t - \frac{2\pi}{3}\right) & -\cos\left(\omega_s t - \frac{2\pi}{3}\right) & 0 \\ -\sin\left(\omega_s t + \frac{2\pi}{3}\right) & -\cos\left(\omega_s t + \frac{2\pi}{3}\right) & 0 \end{bmatrix} \tag{38}$$

$$\begin{aligned} \frac{\mathrm{d}k_r^{-1}}{\mathrm{d}t} &= \Delta\omega\begin{bmatrix} -\sin\Delta\omega t & -\cos\Delta\omega t & 0 \\ -\sin\left(\Delta\omega t - \frac{2\pi}{3}\right) & -\cos\left(\Delta\omega t - \frac{2\pi}{3}\right) & 0 \\ -\sin\left(\Delta\omega t + \frac{2\pi}{3}\right) & -\cos\left(\Delta\omega t + \frac{2\pi}{3}\right) & 0 \end{bmatrix} \\ &= s\omega_s\begin{bmatrix} -\sin\Delta\omega t & -\cos\Delta\omega t & 0 \\ -\sin\left(\Delta\omega t - \frac{2\pi}{3}\right) & -\cos\left(\Delta\omega t - \frac{2\pi}{3}\right) & 0 \\ -\sin\left(\Delta\omega t + \frac{2\pi}{3}\right) & -\cos\left(\Delta\omega t + \frac{2\pi}{3}\right) & 0 \end{bmatrix} \end{aligned} \tag{39}$$

$$k_s\frac{\mathrm{d}k_s^{-1}}{\mathrm{d}t} = \begin{bmatrix} 0 & \omega_s & 0 \\ -\omega_s & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{40}$$

$$k_r\frac{\mathrm{d}k_r^{-1}}{\mathrm{d}t} = \begin{bmatrix} 0 & \Delta\omega & 0 \\ -\Delta\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & s\omega_s & 0 \\ -s\omega_s & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{41}$$

Appendix B. Definition of variables

$$L' = L_{ss} - L_{rs}L_{rr}^{-1}L_{rs}$$

$$= \begin{pmatrix} l_{ss} & 0 \\ 0 & l_{ss} \end{pmatrix} - \begin{pmatrix} l_{rs} & 0 \\ 0 & l_{rs} \end{pmatrix} \begin{bmatrix} \frac{1}{l_{rr}} & 0 \\ 0 & \frac{1}{l_{rr}} \end{bmatrix} \begin{pmatrix} l_{rs} & 0 \\ 0 & l_{rs} \end{pmatrix}$$

$$= \begin{bmatrix} \frac{l_{rr}l_{ss} - l_{rs}^2}{l_{rr}} & 0 \\ 0 & \frac{l_{rr}l_{ss} - l_{rs}^2}{l_{rr}} \end{bmatrix} \tag{42}$$

Taking into account the fact that $l_{rr} = l_2 + l_{rs}$ and $l_{ss} = l_1 + l_{rs}$ the following equation can be written:

$$l' = \frac{l_{rr}l_{ss} - l_{rs}^2}{l_{rr}} = l_1 + \frac{l_2 l_{rs}}{l_2 + l_{rs}} \tag{43}$$

and:

$$X' = l'\omega_s = \left(l_1 + \frac{l_2 l_{rs}}{l_2 + l_{rs}}\right)\omega_s = X_1 + \frac{X_2 X_m}{X_2 + X_m} \tag{44}$$

On the other hand:

$$L = L_{ss} = \begin{pmatrix} l_{ss} & 0 \\ 0 & l_{ss} \end{pmatrix} = \begin{pmatrix} l_1 + l_{rs} & 0 \\ 0 & l_1 + l_{rs} \end{pmatrix} \tag{45}$$

So, the inductance l can be defined as:

$$l = l_{ss} = l_1 + l_{rs} \tag{46}$$

and the reactance X as:

$$X = l\omega_s = (l_1 + l_{rs})\omega_s = X_1 + X_m \tag{47}$$

Appendix C. Equivalence of voltage sources in the dynamic and steady-state model

The equivalence of voltage sources representing the rotor cascade can be deduced from the expressions of stator currents in both models. So, taking the steady-state model into account, the rms stator current can be expressed, according to the references of Fig. 2, as:

$$\bar{I}_s = \frac{(R_R + js(X_2 + X_m))\bar{V} - jX_m \bar{V}_r''}{(R_s + j(X_1 + X_m))(R_R + js(X_2 + X_m)) + sX_m^2} \tag{48}$$

On the other hand, with regard to the dynamic model, by making $d\bar{E}'/dt = 0$ in Eq. (22a) in order to get a steady-state equivalent model, and taking into account Eq. (22b) for substituting the value of $\bar{E}'$ in steady-state as a function of $\bar{V}$ and $\bar{I}_s$, the expression of the rms value for the stator current can be written as:

$$\bar{I}_s = \frac{(R_R + js(X_2 + X_m))\bar{V} - j(X_2 + X_m)\bar{V}_r'}{(R_s + j(X_1 + X_m))(R_R + js(X_2 + X_m)) + sX_m^2} \tag{49}$$

As both expressions must coincide, Eq. (23) must be satisfied.

Appendix D. Parameters of the induction generator used in the simulations

Following the notation given in Fig. 2, the machine used in the simulations has the following parameters, $R_R = 0.00612$ p.u., $R_s = 0.00571$ p.u., $X_1 = 0.06390$ p.u., $X_2 = 0.18781$ p.u., $X_m = 2.78000$ p.u., $H = 3.05$ p.u., gear box ratio, 1:44.38, rotor diameter, 15.2 m, rated power, 350 kW, and rated voltage, 660 V

The power coefficient curve for this machine is given by the equation $c_p = \Sigma_{i=0}^{10} a_i \lambda^i$, where the coefficients a_i are given in Table 1

Table 1
Constants of the polynomial $c_p = f(\lambda)$ for the machine employed in the simulations

Coefficient	Value
a_0	0.0914344959
a_1	−0.486804621
a_2	0.944258742
a_3	−0.909776507
a_4	0.488200324
a_5	−0.153325541
a_6	0.0295642442
a_7	−0.0035602243
a_8	0.000261703947
a_9	−1.07606521e-05
a_{10}	1.8992284e-07

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